Grades 7, 8, and 9 Applied

3 Grade-level Planning Supports

Grade 7

Grade 8

Grade 9

Grades 7, 8, and 9 Applied

3 Grade-level Planning Supports

Grade 7

Grade 7: Content and Reporting Targets

Across the strands and the terms
Problem Solving, Communication, Technology, and Reasoning – expectations to be applied to any/all content clusters.

Term 1 – Content Targets	Term 2 – Content Targets	Term 3 – Content Targets
Number Sense and Numeration*	**Number Sense and Numeration***	**Number Sense and Numeration***
• exponents • multiples and factors • square roots	• adding and subtracting fractions	• integers • order of operations
Measurement*	**Measurement**	**Measurement***
• parallelograms and triangles • rectangular prisms	• problem solving with irregular 2-D shapes • problem solving with 3-D figures	• investigating trapezoids • application of measurement tools (taught and used as contexts for assessment tasks giving assessment data in other strands)
Geometry and Spatial Sense	**Geometry and Spatial Sense***	**Geometry and Spatial Sense***
• building and sketching 3-D models (to support Measurement in this term. Save any assessment data until Term 2 when summative assessment tasks are combined with Measurement)	• congruency	• analysis of tiling and transformational geometry
Patterning and Algebra*	**Patterning and Algebra**	**Patterning and Algebra***
• describing patterns	• solving equations by inspection and systematic trial (introduction - report in all Categories in Term 3)	• apply Patterning and Algebra in Number Sense and Numeration, and Measurement (report on solving equations in all Categories in this term)
	Data Management and Probability*	**Data Management and Probability***
	• importance of data management skills • stem-and-leaf plots and frequency tables • spreadsheet vs. database • central tendency	• sample space for probability experiments • interpret graphs, look for trends and bias • evaluating arguments
Rationale		
Connections between: - multiples, factors, and exponents introduced through applications and perimeter, area, and volume - factors, exponents, perfect squares, and square roots - measurement investigations and all process targets - development of formulas and conceptual understanding of variables in a relationship - building and sketching 3-D models and development of formulas for surface area and volume - measurement concepts and variables and linear equations - describing patterns using words and informal development of understanding of the concept of variable	**Connections between:** - fraction skills and problems involving irregular 2-D shapes and 3-D figures - congruency and tiling and transformational geometry in Term 3 - congruency investigations and data organization - solving equations through systematic trial and application of patterning skills - solving equations by inspection and estimating and using mental math skills - solving equations and reasoning through inspection or systematic trial - importance of data management skills and reasoning - stem-and-leaf plots and frequency tables to measurement and congruency investigations - investigations with data and measures of central tendency	**Connections between:** - integers, whole numbers, decimals - order of operations in formulas and verifying solutions and equations - order of operations and mental math and estimation skills - fractions, exponents, and square roots and cross-stranded investigations/applications - trapezoids and parallelograms and triangles and irregular 2-D shapes - application of measurement tools and consolidation of measurement concepts from Terms 1 and 2 - application of measurement tools and equation solving skills - tiling/transformational geometry and congruency - tiling/transformational geometry and area, perimeter and sketching (Terms 1 and 2) - trends in graphs and patterning - evaluating arguments based on data and measures of central tendency
Leading to: - applications of perfect squares, square roots, factors, exponents (Terms 2 and 3) - understanding of concept of variables (Terms 2 and 3) - using 3-D models to facilitate Measurement (Term 2) - connecting formulas to solving equations (Term 2) - connecting parallelograms and triangles to trapezoids (Term 3)	**Leading to:** - connecting congruency with tiling and transformational geometry (Term 3) - connecting problem solving with irregular 2-D shapes with trapezoids (Term 3) - connecting central tendency with critical analysis of data (Term 3) - connecting irregular 2-D shapes to trapezoids (Term 3) - connecting equation solving to Measurement (Term 3)	**Leading to:** - use of familiar integer manipulatives in Term 1, Grade 8 - multiplication and division of integers (Grade 8) - combining order of operations with fractions (Grade 8)

* Strands for reporting purposes

See Appendix for the clusters of curriculum expectations attached to each of the content targets.

Appendix: Curriculum Expectation Clusters

<table>
<tr><th colspan="3">Grade 7: Number Sense and Numeration</th></tr>
<tr><th>Term 1</th><th>Term 2</th><th>Term 3</th></tr>
<tr><td colspan="3">Across the strands and the terms
Problem Solving, Communication, Technology, and Reasoning – expectations to be applied to any/all content clusters
7m5 • *use estimation to justify or assess the reasonableness of calculations;
7m6 • *solve and explain multi-step problems involving simple fractions, decimals, and percents;
7m7 • *explain, in writing, the process of problem solving using appropriate mathematical language;
7m8 • *use a calculator to solve number questions that are beyond the proficiency expectations for operations using pencil and paper;
7m12 – *explain numerical information in their own words and respond to numerical information in a variety of media;
7m14 – *perform three-step problem solving that involves whole numbers and decimals related to real-life experiences, using calculators;
7m16 – *justify the choice of method for calculations: estimation, mental computation, concrete materials, pencil and paper, algorithms (rules for calculations), or calculators;
7m23 – *ask “what if” questions; pose problems involving simple fractions, decimals, and percents; and investigate solutions;
7m24 – *explain the process used and any conclusions reached in problem solving and investigations;
7m25 – *reflect on learning experiences and describe their understanding using appropriate mathematical language (e.g., in a math journal);
7m26 – *solve problems involving fractions and decimals using the appropriate strategies and calculation methods;
7m27 – *solve problems that involve converting between fractions, decimals, and percents.</td></tr>
<tr><td><u>Exponents</u>
7m4 • understand and explain that exponents represent repeated multiplication;
7m15 – understand that repeated multiplication can be represented as exponents (e.g., in the context of area and volume).
<u>Multiples and Factors</u>
7m11 – generate multiples and factors of given numbers.
<u>Square Roots</u>
7m1 • compare, order, and represent decimals, integers, multiples, factors, and square roots;
7m13 – represent perfect squares and their square roots in a variety of ways (e.g., by using blocks, grids).</td><td><u>Adding and Subtracting Fractions</u>
7m1 • compare, order, and represent decimals, integers, multiples, factors, and square roots;
7m2 • understand and explain operations with fractions using manipulatives;
7m9 – compare and order decimals (e.g., on a number line);
7m17 – demonstrate an understanding of operations with fractions using manipulatives;
7m18 – add and subtract fractions with simple denominators using concrete materials, drawings, and symbols;
7m19 – relate the repeated addition of fractions with simple denominators to the multiplication of a fraction by a whole number (e.g., $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \times \frac{1}{2}$).</td><td><u>Integers</u>
7m10 – compare and order integers (e.g., on a number line);
7m21 – represent the addition and subtraction of integers using concrete materials, drawings, and symbols;
7m22 – add integers, with and without the use of manipulatives.
<u>Order of Operations</u>
7m3 • demonstrate an understanding of the order of operations with brackets;
7m20 – demonstrate an understanding of the order of operations with brackets and apply the order of operations in evaluating expressions that involve whole numbers and decimals.</td></tr>
</table>

* Expectations that require that students be given the opportunity to learn through inquiry. Learning through problem solving is recommended for most other curriculum expectations.

Overall curriculum expectations are designated by the • after the number.
Specific curriculum expectations are designated by the – after the number.

Appendix: Curriculum Expectation Clusters

<table>
<tr><th colspan="3">Grade 7: Measurement</th></tr>
<tr><th>Term 1</th><th>Term 2</th><th>Term 3</th></tr>
<tr><td colspan="3">Cross-Strand and Cross-Term:
Problem Solving, Communication, Technology, and Reasoning - expectations to be applied to any/all content clusters
7m28 • *demonstrate a verbal and written understanding of and ability to apply accurate measurement strategies that relate to their environment;
7m29 • *identify relationships between and among measurement concepts (linear, square, cubic, temporal, monetary);
7m32 – *create definitions of measurement concepts;
7m33 – *describe measurement concepts using appropriate measurement vocabulary;
7m35 – *make increasingly more informed and accurate measurement estimations based on an understanding of formulas and the results of investigations.</td></tr>
<tr><td><u>Parallelograms and Triangles</u>
7m41 – *develop the formulas for finding the area of a parallelogram and the area of a triangle.
<u>Rectangular Prisms</u>
7m42 – *develop the formula for finding the surface area of a rectangular prism using nets;
7m43 – *develop the formula for finding the volume of a rectangular prism using concrete materials;
7m44 – understand the relationship between the dimensions and the volume of a rectangular prism.</td><td><u>Problem Solving with Irregular 2-D Shapes</u>
7m30 • solve problems related to the calculation and comparison of the perimeter and the area of irregular two-dimensional shapes;
7m36 – understand that irregular two-dimensional shapes can be decomposed into simple two-dimensional shapes to find the area and perimeter;
7m37 – estimate and calculate the perimeter and area of an irregular two-dimensional shape.
<u>Problem Solving with 3-D Figures</u>
7m31 • apply volume formulas to problem-solving situations involving rectangular prisms;
7m45 – calculate the surface area and the volume of a rectangular prism in a problem-solving context;
7m46 – sketch a rectangular prism given its volume.</td><td><u>Investigating Trapezoids</u>
7m38 – *develop the formula for finding the area of a trapezoid;
7m39 – estimate and calculate the area of a trapezoid, using a formula;
7m40 – draw a trapezoid given its area and/or perimeter.
<u>Application of Measurement Tools</u>
7m34 – research and report on the uses of measurement instruments in projects at home, in the workplace, and in the community.</td></tr>
</table>

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Appendix: Curriculum Expectation Clusters

Grade 7: Geometry and Spatial Sense		
Cross-Strand and Cross-Term: **Problem Solving, Communication, Technology, and Reasoning - expectations to be applied to any/all content clusters** 7m47 • *identify, describe, compare and classify geometric figures; 7m50 • *explore transformations of geometric figures; 7m52 • *use mathematical language effectively to describe geometric concepts, reasoning, and investigations; 7m61 – *recognize the image of a two-dimensional shape under a translation, a reflection, and a rotation in a variety of contexts.		
Term 1	**Term 2**	**Term 3**
Building and Sketching 3-D Models 7m48 • identify, draw, and construct three-dimensional geometric figures from nets; 7m53 – recognize the front, side, and back views of three-dimensional figures; 7m54 – sketch front, top, and side views of three-dimensional figures with or without the use of a computer application; 7m55 – sketch three-dimensional objects from models and drawings; 7m56 – build three-dimensional figures and objects from nets.	**Congruency** 7m49 • identify congruent and similar figures; 7m59 – *identify through investigation the conditions that make two shapes congruent; 7m58 – explain why two shapes are congruent; 7m57 – identify two-dimensional shapes that meet certain criteria (e.g., an isosceles triangle with a 40° angle); 7m60 – create and solve problems involving the congruence of shapes; 7m64 – construct and analyse tiling patterns with congruent tiles.	**Analysis of Transformational Geometry** 7m51 • understand, apply, and analyse key concepts in transformational geometry using concrete materials and drawings; 7m62 – create and analyse designs that include translated, rotated, and reflected two-dimensional images using concrete materials and drawings, and using appropriate computer applications; 7m63 – identify whether a figure will tile a plane; 7m65 – describe designs in terms of images that are congruent, translated, rotated, and reflected.

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Appendix: Curriculum Expectation Clusters

Grade 7: Patterning and Algebra		
Analysis of Transformational Geometry 7m51 • *understand, apply, and analyse key concepts in transformational geometry using concrete materials and drawings; 7m62 – *create and analyse designs that include translated, rotated, and reflected two-dimensional images using concrete materials and drawings, and using appropriate computer applications; 7m63 – *identify whether a figure will tile a plane; 7m64 – *construct and analyse tiling patterns with congruent tiles; 7m65 – *describe designs in terms of images that are congruent, translated, rotated, and reflected.		
Term 1	**Term 2**	**Term 3**
Describing Patterns 7m66 • identify the relationships between whole numbers and variables; 7m67 • identify, extend, create, and discuss patterns using whole numbers and variables; 7m68 • identify, create, and solve simple algebraic equations; 7m73 – interpret a variable as a symbol that may be replaced by a given set of numbers; 7m74 – write statements to interpret simple formulas; 7m76 – evaluate simple algebraic expressions by substituting natural numbers for the variables; 7m77 – translate simple statements into algebraic expressions or equations.	**Solving Equations by Inspection and Systematic Trial** 7m78 – solve equations of the form $ax = c$ and $ax + b = c$ by inspection and systematic trial, using whole numbers, with and without the use of a calculator; 7m79 – solve problems giving rise to first-degree equations with one variable by inspection or by systematic trial; 7m80 – establish that a solution to an equation makes the equation true (limit to equations with one variable); 7m98 – use conventional symbols, titles, and labels when displaying data. *Reinforce and practice Linear Equations by applying them in Other Strands*	*Reinforce and practice Linear Equations by applying them in Other Strands*

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Appendix: Curriculum Expectation Clusters

Grade 7: Data Management and Probability

Cross-Strand and Cross-Term:
Problem Solving, Communication, Technology, and Reasoning - expectations to be applied to any/all content clusters

7m81 • *systematically collect, organize, and analyse data;

7m83 • *use computer applications to examine and interpret data in a variety of ways;

7m85 • *construct graphic organizers using computer applications;

7m86 • *interpret displays of data and present the information using mathematical terms;

7m87 • *evaluate data and make conclusions from the analysis of data;

7m88 • *use and apply a knowledge of probability;

7m98 – *use conventional symbols, titles, and labels when displaying data;

7m102 – *display data on bar graphs, pictographs, and circle graphs, with and without the help of technology;

7m103 – *make inferences and convincing arguments that are based on data analysis (e.g., use census information to predict whether the population in Canada will increase).

Term 1	Term 2	Term 3
	Importance of Data Management Skills 7m82 • recognize the different levels of data collection; 7m84 • develop an appreciation for statistical methods as powerful means of decision making; 7m89 – demonstrate the pervasive use of data and probability; 7m90 – understand the impact that statistical methods have on decision making. **Stem-and-leaf Plots and Frequency Tables** 7m91 – collect and organize data on tally charts and stem-and-leaf plots, and display data on frequency tables, using simple data collected by the students (primary data) and more complex data collected by someone else (secondary data); 7m92 – understand how tally charts and frequency tables can be used to record data. **Spreadsheet vs. Database** 7m93 – understand the difference between a spreadsheet and a database for recording and retrieving information; 7m94 – *search databases for information and interpret the numerical data. **Compare and Contrast Measures of Central Tendency** 7m95 – understand that each measure of central tendency (mean, median, mode) gives different information about the data; 7m101 – describe data using calculations of mean, median, and mode.	**Sample Space for Probability Experiments** 7m106 – *develop intuitive concepts of probability and understand how probability can relate to sports and games of chance; 7m107 – list the possible outcomes of simple experiments by using tree diagrams, modelling, and lists; 7m108 – identify the favourable outcomes among the total number of possible outcomes and state the associated probability (e.g., of getting a heads in a fair coin toss); 7m109 – apply a knowledge of probability in sports and games of chance. **Interpreting Graphs, Looking for Trends and Bias** 7m96 – identify and describe trends in graphs, using informal language to identify growth, clustering, and simple attributes (e.g., line graphs that level off); 7m97 – describe in their own words information presented on tally charts, stem-and-leaf plots, and frequency tables; 7m99 – analyse bias in data-collection methods; 7m100 – read and report information about data presented on bar graphs, pictographs, and circle graphs, and use the information to solve problems. **Evaluate Arguments** 7m104 – evaluate arguments that are based on data analysis; 7m105 – *explore with technology to find the best presentation of data.

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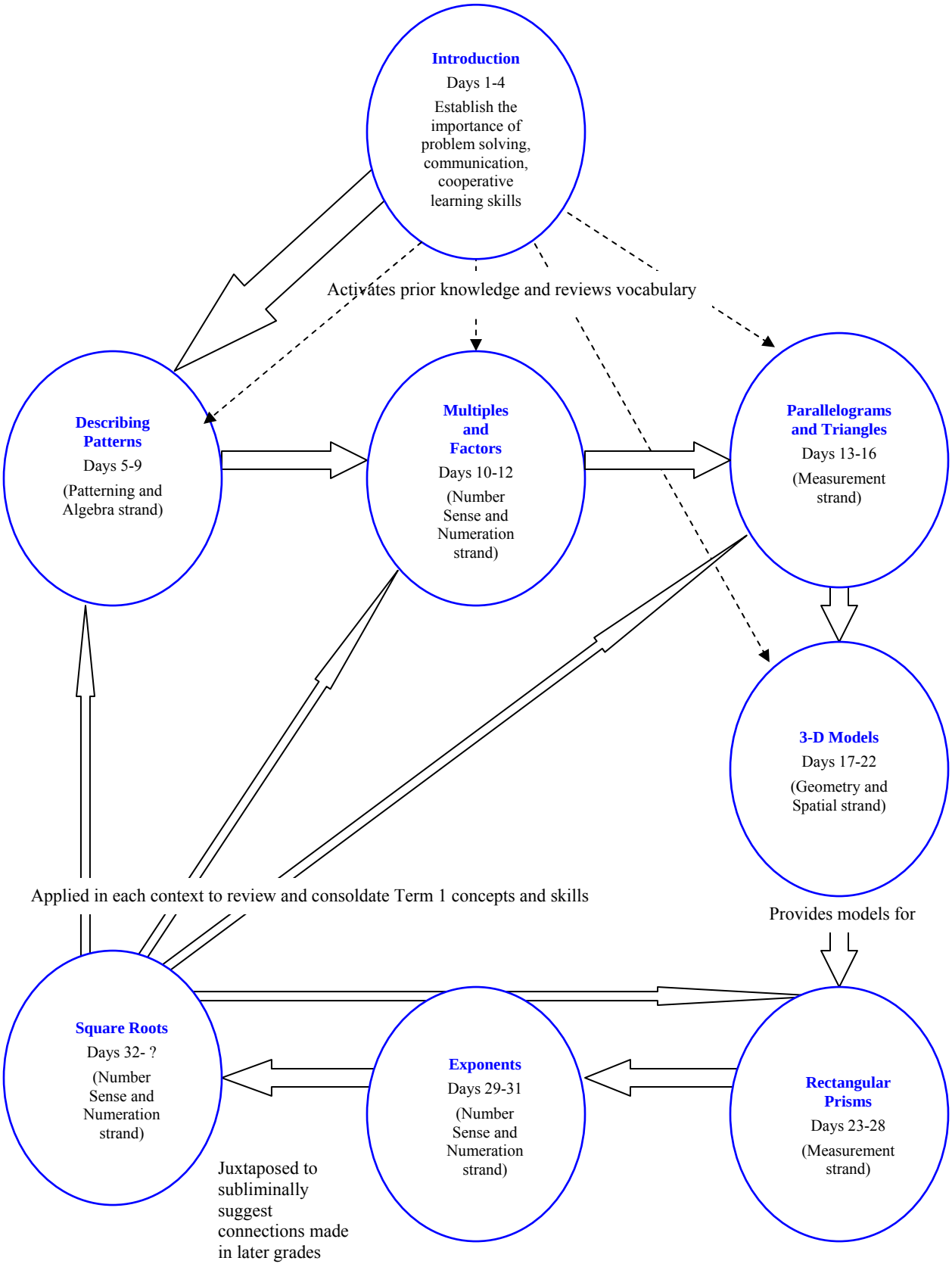
Grade 7: Term 1 Content Flow



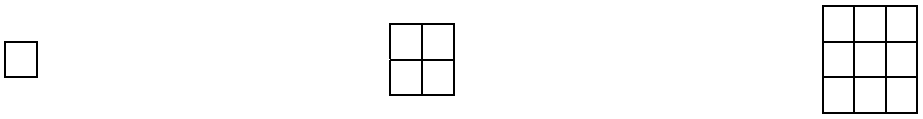
Other sequences are possible.
Suggestions for development of Term 1 Square Roots lessons are included on pages 8-11.

Suggestions

- Use the theme of square roots to review earlier Term 1 curriculum clusters in preparation for summative assessments based on the entire term's content.
- Use the theme of square roots as a segue to the Term 2 cluster — Solving Equations.

Examples

1. Connect square roots to the Describing Patterns cluster.



a) Refer to the pattern shown to complete the chart and extend the pattern on the chart.

Number of 1-unit squares in the large square	Side length of the large square
1	1
4	2
9	

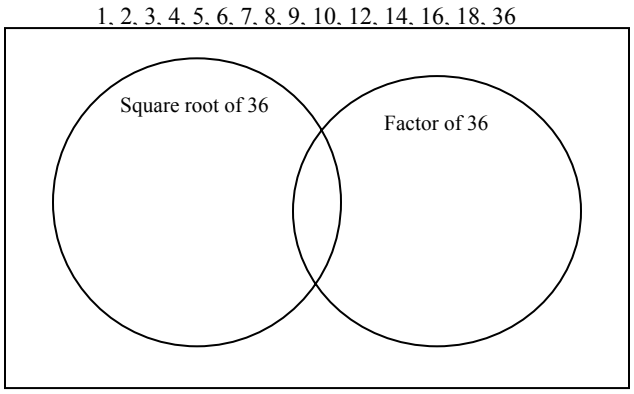
b) Describe the pattern.

c) Use the pattern to predict the side length of the large square comprised of:
   i) 100 1-unit squares
   ii) 225 1-unit squares.

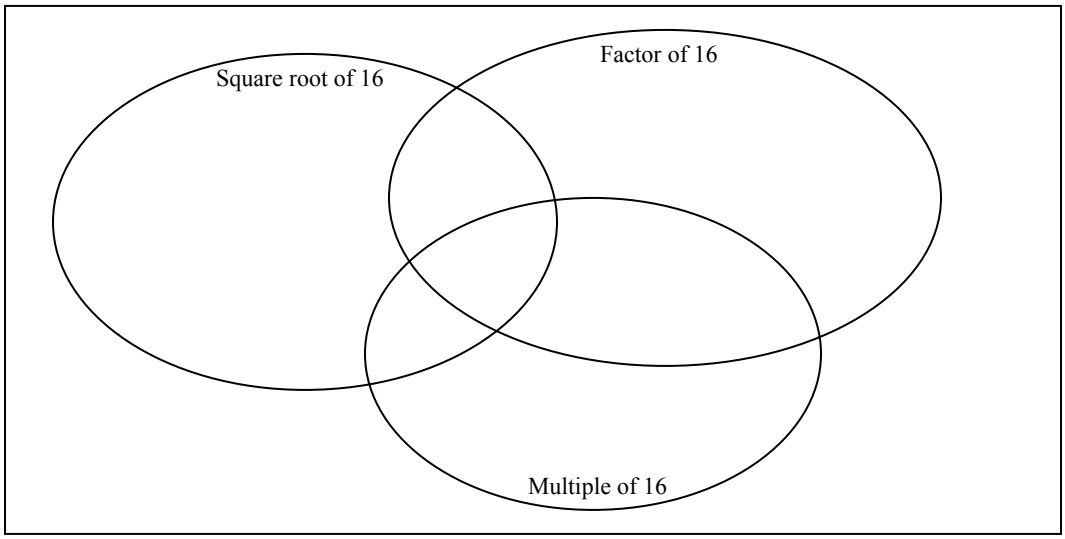
d) Explain why the numbers 5 and 1000 will not appear in the 1st column.

2. Connect square roots to multiples and factors.

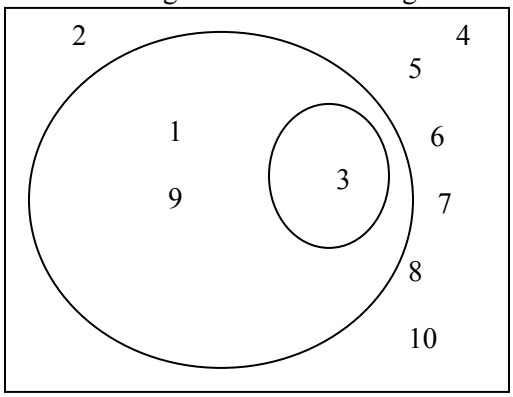
a) Complete the Venn diagram.
b) Re-draw the Venn diagram to better show the relationships among these sets of numbers. (Hint: If part of the diagram is empty, that part of the diagram does not need to be included.)

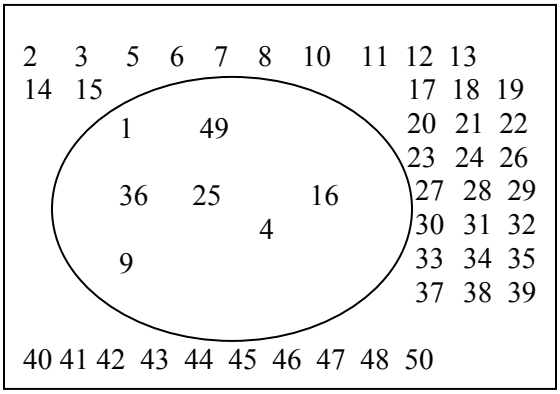


c) Complete the Venn diagram:



d) Label each region in the Venn diagrams:





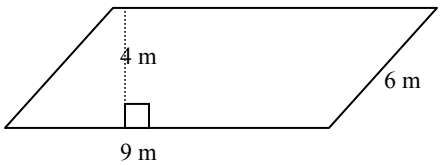
e) Explain why the numbers 64, 81, and 100 are called perfect squares, while the numbers 65, 80 and 99 are not perfect squares.

f) Is it possible to construct a square that has an area of exactly 65 cm^2?
Explain why or why not and include a diagram.

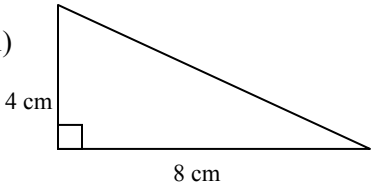
3. Connect square roots to squares, parallelograms, and triangles.
a) What would be the side length of a square that has the same area as each of the following?

(Note: Diagrams are not drawn to scale.)

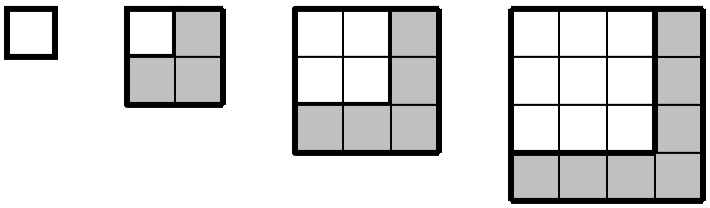
i)



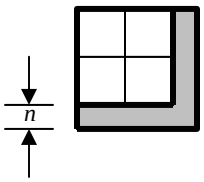
ii)



b) A square can grow in size by having successive L-shaped additions built around the right side and bottom as illustrated below.



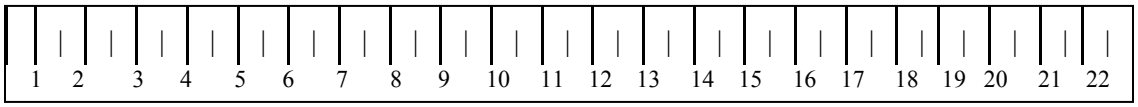
Areas of these squares are represented by perfect square numbers __, __, __, and__units^2. Lengths of sides of the squares are the square roots of 1, 4, 9, and 16, or __, __, __, and __ units respectively.
A thin L-shaped addition is placed around the 2nd diagram above as shown below.



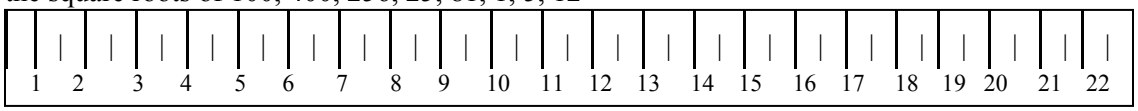
i) What do you know about the area of this new square?
ii) What do you know about the length of each side of the new square?
iii) If n is 0.5 unit, what are the exact area and side length of the new square?
iv) Is it possible to find some value for n that would result in an exact area of 5 units^2 for the new square? Explain why or why not.

4. Compare and order decimals, integers, multiples, factors, and square roots.

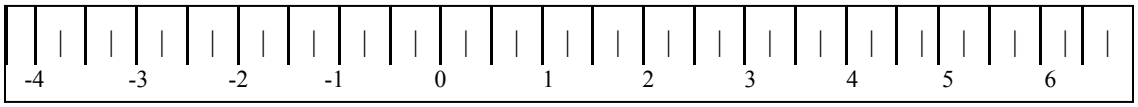
a) Mark with an X where the following numbers would be found on the ruler:
i) all factors and multiples of 6 that are less than 20



ii) the square roots of 100, 400, 256, 25, 81, 1, 5, 12



iii) the following numbers: 2, $\sqrt{2}$, −2, 2^2



5. Connect square roots to rectangular prisms.

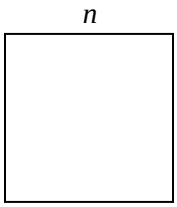
   a) If the numbers on opposite sides of a number cube are the square or the square root of each other, what is the minimum possible sum of all sides of the number cube?

   b) What could be the whole number dimensions of a rectangular prism:
      i) whose length is the square of the width and height is the square root of the width?
      ii) whose ends are square and whose volume is 80 m^3? 81 m^3?
      iii) whose ends are square, and whose total surface area is 418 cm^2, and whose sides have a total surface area of 176 cm^2?

6. Connect square roots to solving equations by inspection and systematic trial in term 2.

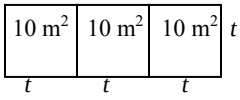
   a) What geometry question has the answer $3 \times 3 = 9$?

   b) If the area of a square is 16, what equation can you write involving s, the length of a side of the square?

c) 

If the area of this square is 144 cm^2, what equation can be used to represent this situation? Solve the equation for n.

   d) Joel made several trials to solve the following problem using the equation $3t^2 = 30$.



   He organized his work in a chart as shown. Using Joel's data, what do you know about a solution to this equation as Joel completes each row? Find a value for t that yields a value closer to 30 than 30.72. Enter your work in the empty rows of the table.

t	$3t^2$	**What I know about the solution to $3t^2=30$**
3	$3\times3^2=3\times9=27$	
4	$3\times4^2=3\times16=48$	
3.5	$3\times3.5^2=3\times12.25=36.75$	
3.2	$3\times3.2^2=3\times10.24=30.72$	

Interpreting the Lesson Outline Template

Lesson Outline: Days 5 - 9

Grade 7

Sequence of Lessons Addressing a Theme

Grade Level

BIG PICTURE

Students will:

- explore and generalize patterns;
- develop an understanding of variables;
- investigate and compare different representations of patterns.

Lessons are planned to help students develop and demonstrate the skills and knowledge detailed in the curriculum expectations.

- **To help students value and remember the mathematics they learn, each lesson is connected to and focussed on important mathematics as described in the Big Picture.**
- **Since students need to be active to develop understanding of these larger ideas, each point begins with a verb.**
- **Sample starter verbs: represent, relate, investigate, generate, explore, develop, design, create, connect, apply**

Day	Lesson Title	Description	Expectations
5	Toothpick Patterns	• Review patterning concepts • Build a growing pattern • Explore multiple representations	7m70, 7m72 CGE 3c, 4f
6	Patterns with Tiles	• Build a pattern • Introduce the *n*th term	7m66, 7m71 CGE 4b
7	Pattern Practice	• Continued development of patterning skills	7m67, 7m71, 7m75 CGE 2c, 5e
8	Pattern Exchange	• Class sharing of work from previous day.	7m69, 7m75 CGE 2c, 5e
9	Performance Task	• Performance Task - individual	7m66, 7m67, 7m73, 7m75 CGE 5g

A brief descriptive lesson title

List curriculum expectations (and CGEs) by code

- **Two or three points to describe the content of this lesson.**
- **Points begin with a verb.**
- **Individual lesson plans elaborate on how objectives are met.**

NOTES

a) While planning lessons, teachers must judge whether or not pre-made activities support development of big ideas and provide opportunities for students to understand and communicate connections to the "Big Picture."

b) Ontario Catholic School Graduation Expectations (CGEs) are included for use by teachers in Catholic schools.

c) Consider auditory, kinesthetic, and visual learners in the planning process and create lessons that allow students to learn in different ways.

d) The number of lessons in a group will vary.

e) Schools vary in the amount of time allocated to the mathematics program. The time clock/circle on completed Grade 7 and 8 lessons suggests the fractions of the class to spend on the Minds On, Action!, and Consolidate/Debrief portions of the class. Grade 9 Applied lessons are based on 75-minute classes.

f) Although some assessment is suggested during each lesson, the assessment is often meant to inform adjustments the teacher will make to later parts of the lesson or to future lessons. A variety of more formal assessments of student achievement could be added.

Download the Lesson Outline Template at www.curriculum.org/occ/tips/downloads.shtml

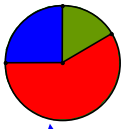
Interpreting the Lesson Planning Template

Day 1: Encouraging Others

Day #: Lesson Title

Grade level

Grade 8



Description
- Practise the social skill of encouraging others.
- Identify strategies involving estimation problems.
- Set the stage for using estimation as a problem-solving strategy.

Same two or three objectives listed in the lesson outline

Materials
- BLM 1.1
- birdseed

Materials used in the lesson

Time colour-coded to the three parts of the day's lesson

Suggested student grouping → teaching/learning strategy for the activity.

Assessment Opportunities

Minds On ...

Whole Group → Brainstorm

Explain why it is important to encourage others. Explicitly teach the social skill, "Encouraging Others," through a group brainstorm. Create an anchor chart using the criteria: What does it look like? What does it sound like?

"Learning is socially constructed; we seldom learn isolated from others." Bennett & Rolheiser

- Mentally engages students at start of class
- Makes connections between different math strands, previous lessons or groups of lessons, students' interests, jobs, etc.
- Introduces a problem or a motivating activity - orients students to an activity or materials.

Action!

Think/Pair/Share → Gather Data

Use an overhead of the Think/Pair/Share process (TIP 2.1) and student copies of BLM 1.1. Students gather data.

Learning Skill/Observation/Mental Note: Circulate, observing social skills and listening to students.

Consider using stickers as a recognition for examples of the social skill being applied by a group

Indicates an assessment opportunity - what is assessed/strategy/scoring tool

Share with students some of the positive words and actions observed during the activity and invite students to make additions to the anchor chart on Encouraging Others.

Indicates suggested assessment

Whole Class → Sharing

Based on 'teachable topics' during the Think/Pair/Share Activity, e.g., a particularly effective phrase/statement expressed by a student, clarification of the cooperative learning strategy, an interesting result on BLM 1.1, ask representatives of groups to share their results or report on their process.

Solving Fermi problems is a way to collect diagnostic assessment data about social skills, academic understandings and attitudes towards mathematics (TIP 1.2).

- Students do mathematics: reflecting, discussing, observing, investigating, exploring, creating, listening, reasoning, making connections, demonstrating understanding, discovering, hypothesizing
- Teachers listen, observe, respond and prompt

Consolidate Debrief

Whole Class → Discussion

Use the posters Inquiry Model Flow Chart, Problem-Solving Strategies, and Understanding the Problem. Discuss how these posters will be of assistance over the next few days as well as during the whole math program. Point out that when students encourage others, it makes it safe for them to try new things and contribute to group activities.

- "Pulls out´ the math of the activities and investigations
- Prepares students for Home/Further Classroom Consolidation

Tips for the Teacher
These include:
- instructional hints
- explanations
- background
- references to resources
- sample responses to questions/tasks

Home Activity or Further Classroom Consolidation

Interview one or more adults about estimation using the following guiding questions and record your responses in a math journal. Summarize what you notice about the responses. You may be asked to share this math journal entry with the class.

Meaningful and appropriate follow-up to the lesson.

Social Skill Practice Reflection

Focus for the follow-up activity

Download the Lesson Planning Template at www.curriculum.org/occ/tips/downloads.shtm

Lesson Outline: Days 1 - 4

Grade 7

BIG PICTURE

Students will:

- recognize the importance of problem solving and investigation in learning mathematics;
- practise appropriate learning skills, e.g., listening to classmates, legitimizing errors as part of the learning process, tolerating ambiguity, demonstrating perseverance;
- demonstrate understanding of key mathematics vocabulary and processes; participate in activities that touch on each of the five strands of mathematics.

Day	Lesson Title	Description	Expectations
1	Celebrating Seven	• Investigate patterns involving the number 7. • Investigate patterns involving prime numbers and perfect squares.	7m8, 7m25, 7m26, 7m27, 7m69, 7m71, 7m81 CGE 3c, 4a, 4e
2	Tangram Tune-Up	• Review geometric language. • Introduce new geometric terminology. • Construct tangram pieces and create 2-D composite shapes.	7m25, 7m26, 7m57, 7m60, 7m83 CGE 2c, 4f
3	Summer Survival	• Develop survey questions. • Display data.	7m25, 7m71, 7m81, 7m85, 7m102 CGE 2c, 5a
4	Can-Do Puzzles	• Participate in a variety of tasks which involve problem solving, patterning, performing calculations, working with 2-D shapes and 3-D models.	7m7, 7m25, 7m27, 7m57, 7m60, 7m69, 7m71, 7m83 CGE 3c, 5a, 5e

Description

- Investigate patterns involving the number 7.
- Investigate patterns involving prime numbers and perfect squares.

Materials

- calculators
- BLM 1.1, 1.2, 1.3
- coloured markers

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Briefly identify the key messages about how your Grade 7 learning community will operate. Review the succinct messages for ideas and posters. Provide opportunities for students to suggest "why" the rules or procedures are necessary/appropriate. Set the stage for the development of positive attitudes toward mathematics.

Whole Class → Brainstorm

Where do we find 7 in our world?

Action!

Pairs → Investigation

Curriculum Expectations/Observation/Checkbric: Observe students as they complete the activity, watching their calculator use and their patterning skills.

Pairs of students use a calculator and BLM 1.1 to discover patterns involving the number 7. Could there be more than 6 digits in the period of these decimals? Explain.

Create other '7-related' patterns or statements, using your calculator.

Clarify the term "perfect square" for students – linking it to measurement activities from previous years.

Students complete BLM 1.2 and record the solutions on overheads using different colours. Overlay the overheads for students to observe the overall patterns.

Note: Calculators may not display the repeating decimals as they may round the last digit or display fewer than 8 digits.

To encourage quick recall of previously learned material, individual students might make a list – "Number facts about 7 that I know."

Whole Class → Student Presentations

Students report on their findings from BLM 1.1 and 1.2. Encourage oral or written presentations using precise language, diagrams, and charts. Model appropriate ways to discuss errors or consider alternate descriptions.

Select students to add vocabulary to the Word Wall. TIP 10

Consolidate Debrief

Individual → Response Journal

Provide sentence stems that allow students to summarize today's activities, e.g., I learned…, I discovered…, I remembered…, Our class will be great if we all…, I wonder why….

Home Activity or Further Classroom Consolidation

BLM 1.4, Wonders of the World

Exploration

Explore these topics at the school or community library or at a website:

1. Investigate the Seven Wonders of the World, using worksheet 1.3. (There are ancient, modern, and natural wonders.)
2. Since 'septem' means 7, why is September the 9th month of the year?
3. What is a septogenarian? Make a list of septogenarians you know.

1.1: Celebrating Seven

Name:
Date:

- Use a calculator to complete each of the following tasks.
- Identify any patterns you notice.
- Use your patterns to predict the next three answers in the pattern.
- Check using your calculator.

1. Convert each of the following fractions to decimal form. Identify the sequence of digits that occurs repeatedly within each decimal. Count the number of digits in the sequence.

   $\frac{1}{7}$ $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{6}{7}$

2. Find the product of 12 345 679 and 63

3. Find the values of 7×7, $7 \times 7 \times 7$, $7 \times 7 \times 7 \times 7$, … . Analyse the pattern and extend it without a calculator. Check using a calculator.

4. Find the values of 7×7, 7×77, 7×777, … . Analyse the pattern and extend it without a calculator. Check using a calculator.

5. Calculate each of the following products. Analyse the pattern and extend it without a calculator. Check using a calculator.
   $15\,873 \times 7$
   $15\,873 \times 14$
   $15\,873 \times 21$

1.2: Prime Patterns

Name:
Date:

Complete the spiral of numbers and then shade in each square that shows a prime. Using a different colour, shade in all of the perfect squares.
Record all the patterns you notice about prime and composite numbers.

			16	**15**	**14**	**13**			
			5	**4**	**3**	**12**			
			6	**1**	**2**	**11**			
			7	**8**	**9**	**10**			

1.2: Prime Patterns – Prime (continued)

(Answers)

100	99	98	97	96	95	94	96	92	91
65	64	63	62	61	60	59	58	57	90
66	37	36	35	34	33	32	31	56	89
67	38	17	16	15	14	13	30	55	88
68	39	18	5	4	3	12	29	54	87
69	40	19	6	1	2	11	28	53	86
70	41	20	7	8	9	10	27	52	85
71	42	21	22	23	24	25	26	51	84
72	43	44	45	46	47	48	49	50	83
73	74	75	76	77	78	79	80	81	82

1.2: Prime Patterns - Perfect Squares (continued)

(Answers)

100	99	98	97	96	95	94	96	92	91
65	64	63	62	61	60	59	58	57	90
66	37	36	35	34	33	32	31	56	89
67	38	17	16	15	14	13	30	55	88
68	39	18	5	4	3	12	29	54	87
69	40	19	6	1	2	11	28	53	86
70	41	20	7	8	9	10	27	52	85
71	42	21	22	23	24	25	26	51	84
72	43	44	45	46	47	48	49	50	83
73	74	75	76	77	78	79	80	81	82

1.3: Seven Wonders of the Ancient World

Using the Internet or other available reference material, find information about the Seven Wonders of the Ancient World.

1. In your opinion, which wonders are “mathematical”?

2. Which two wonders would you most like to visit? Explain why.

3. Why do you think only seven wonders were included in the original list?

4. What “wonders” have been added to the list?

5. What are the “natural” wonders of the world?

6. What other lists of seven items can you find?

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6. What other lists of seven items can you find?

1.4: The Wonders of the World – Teacher Reference

The Seven Natural Wonders of the World

- Mount Everest
- The Great Barrier Reef
- The Grand Canyon
- Victoria Falls
- The Harbour of Rio de Janeiro
- Paricutin Volcano
- The Northern Lights

The Seven Wonders of the Ancient World

- The Great Pyramid of Giza
- The Hanging Gardens of Babylon
- The Temple of Artemis at Ephesus
- The Statue of Zeus at Olympia
- The Mausoleum at Halicarnassus
- The Colossus of Rhodes
- The Pharos of Alexandria

The Seven Wonders of the Modern World

- The Empire State Building
- The Itaipú Dam
- The CN Tower
- The Panama Canal
- The Channel Tunnel
- The North Sea Protection Works
- The Golden Gate Bridge

Description

- Review geometric language.
- Introduce new geometric terminology.
- Construct tangram pieces and create 2-D composite shapes.

Materials

- square paper
- BLM 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8
- Transparency of BLM 2.1
- tangram set

Assessment Opportunities

Minds On ...

Whole Class → Reflection

Ask several students to share their math journal entries. Show students the 7 pieces of a tangram set.

Provide silhouettes of tangram figures on the overhead for the following activity (BLM 2.7). This activity serves as an introduction to new vocabulary (midpoint, perpendicular, bisector).

To activate prior knowledge, use as much rich geometry, fraction, and measurement vocabulary as possible.

Action!

Whole Class → Model Making

Using a transparency of BLM 2.1, model the procedure for making the tangram pieces. Students make their own tangram pieces, using plain square paper as they follow the modelling.

1. After making the first cut ask, What shapes have been made?
2. After the second cut ask, How are these triangles the same as the larger triangle?
3. Make the third cut along the fold between the midpoints of the shorter sides of the remaining large triangle. Ask, What kind of shapes, lines and angles do you see?
4. Cut along the perpendicular bisector (lines of symmetry) of the trapezoid. Ask: How are these smaller trapezoids different from the larger one?
5. Continue with steps 5 and 6 on BLM 2.1, asking about the shapes as they are cut out and labelled.

Scaffold: Provide a tangram ready to be cut out and ask students to describe the pieces as they are cut out. (BLM 2.2)

Provide envelopes for students to store their tangram pieces for future lessons (Day 15).

Small Groups → Discussion

Complete selected activities from BLM 2.3 and 2.4. Have students describe the shapes using precise vocabulary. Show solutions on an overhead (BLM 2.5).

A propped-up textbook between partners can serve as a screen.

Pairs → Game

Students work with a partner. Partner A creates a figure using the tangram pieces, and then provides instructions so that Partner B can recreate the figure with another tangram set. Partner B may not ask questions for clarification, but may only respond to Partner A's directions. When finished, students compare the two figures and assess the quality of Partner A's instructions. The partners exchange roles and then work together on BLM 2.6.

Note: BLM 2.8, Area Tangrams and BLM 2.7, Tangram Puzzles.

Curriculum Expectations/Observation/Rubric: Focus on fluent, accurate, and effective use of mathematics vocabulary.

Consolidate Debrief

Whole Class → Note Making

List the mathematics terminology used in class today. Draw a sketch to show the meaning of each term.

Select a student to add vocabulary to the Word Wall.

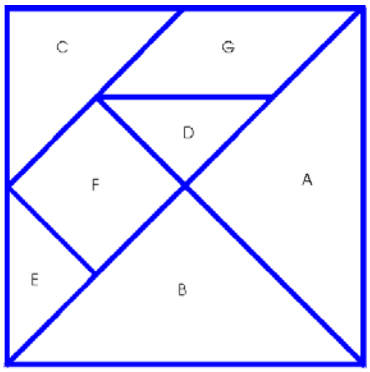
Exploration

Home Activity or Further Classroom Consolidation

Challenge someone at home or in class with tangram puzzles. Ask if they know any paper folding tricks or activities. Practise the activities to show the class the next day.

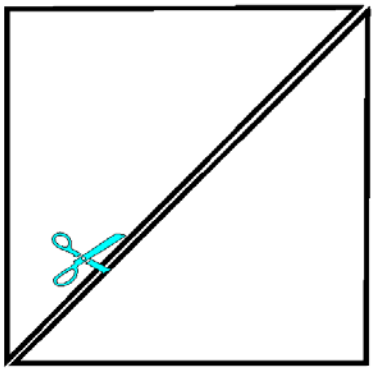
Research using the Internet to find other tangram activities.

2.1: Constructing a Tangram from a Square

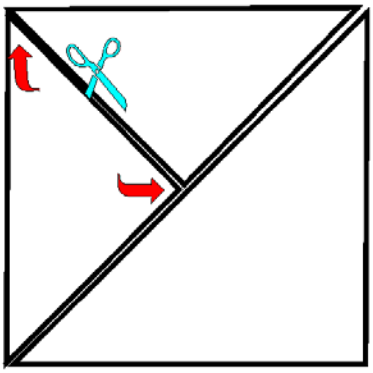


The seven tangram pieces can be geometrically constructed from a square.

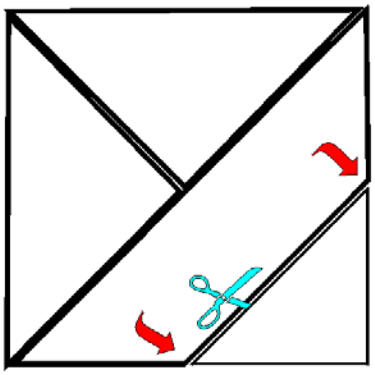
Use paper folding and scissors to cut along the folds and create the seven tangram pieces.



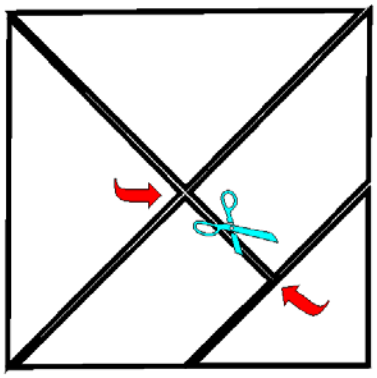
Fold along the diagonal of the square. Cut.



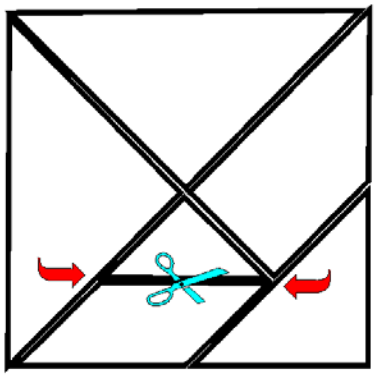
Fold the perpendicular bisector of one of the right isosceles triangles. Cut along the bisector.



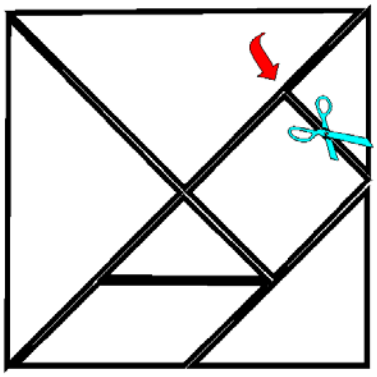
Crease the midpoints of the two sides of the large right isosceles triangle. Fold the line joining the midpoints. Cut.



Crease the midpoints of the two parallel sides of the trapezoid. Cut along the line that joins the midpoints.

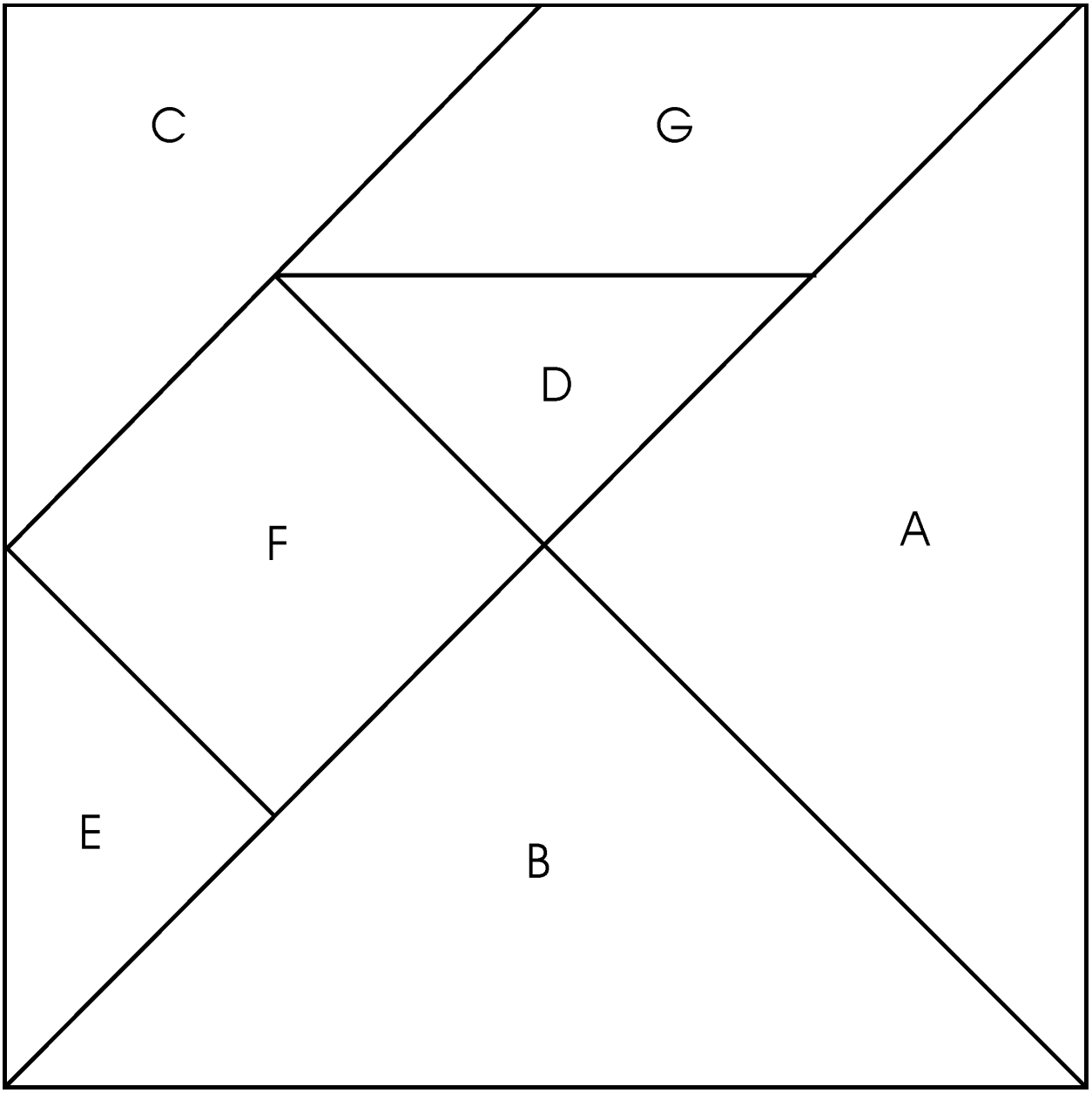


Crease the midpoint of the longest side of the right trapezoid. Fold a line joining the midpoint to the opposite vertex. Cut.



Crease the midpoint of the longest side of the trapezoid. Fold an altitude from the midpoint. Cut.

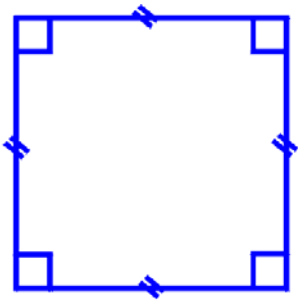
2.1: Constructing a Tangram from a Square – Pattern
(continued)



2.3: Creating Squares Using Tangram Pieces

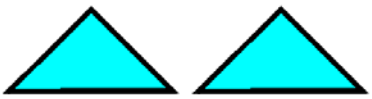
Name:
Date:

A square has four equal sides and four 90° angles.



Use the following tangram pieces to create squares:

a) 

b) 

c) 

d) 

e) 

f) 

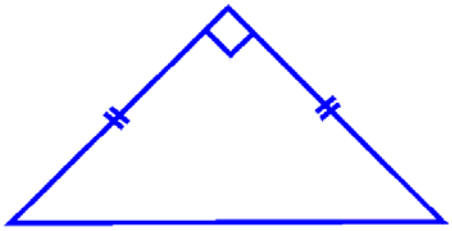
g) 

h) Use all 7 pieces.

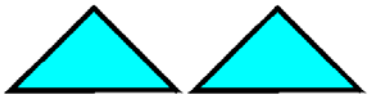
2.4: Creating Right-Angled Isosceles Triangles Using Tangram Pieces

Name:
Date:

A right isosceles triangle has two equal sides with an enclosed right angle.



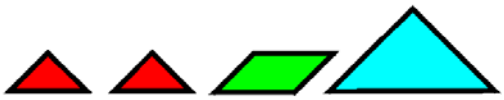
Use the following tangram pieces to create right-angled isosceles triangles.

a) 

b) 

c) 

d) 

e) 

f) 

g) 

h) 

i) 

j) 

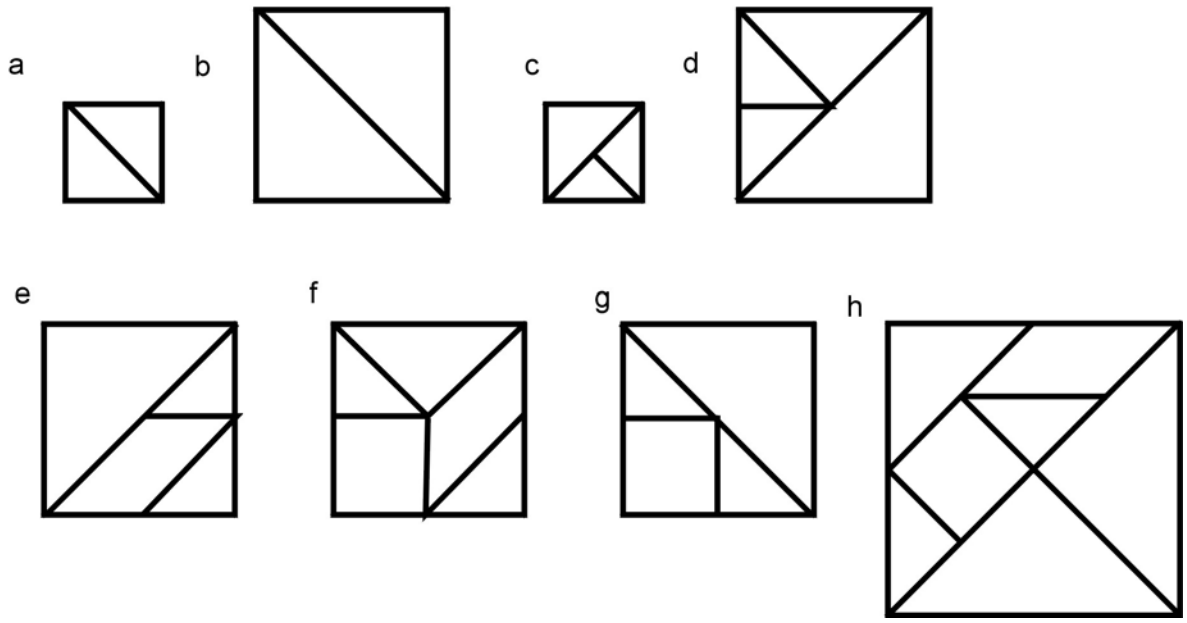
k) 

l) Use all 7 pieces.

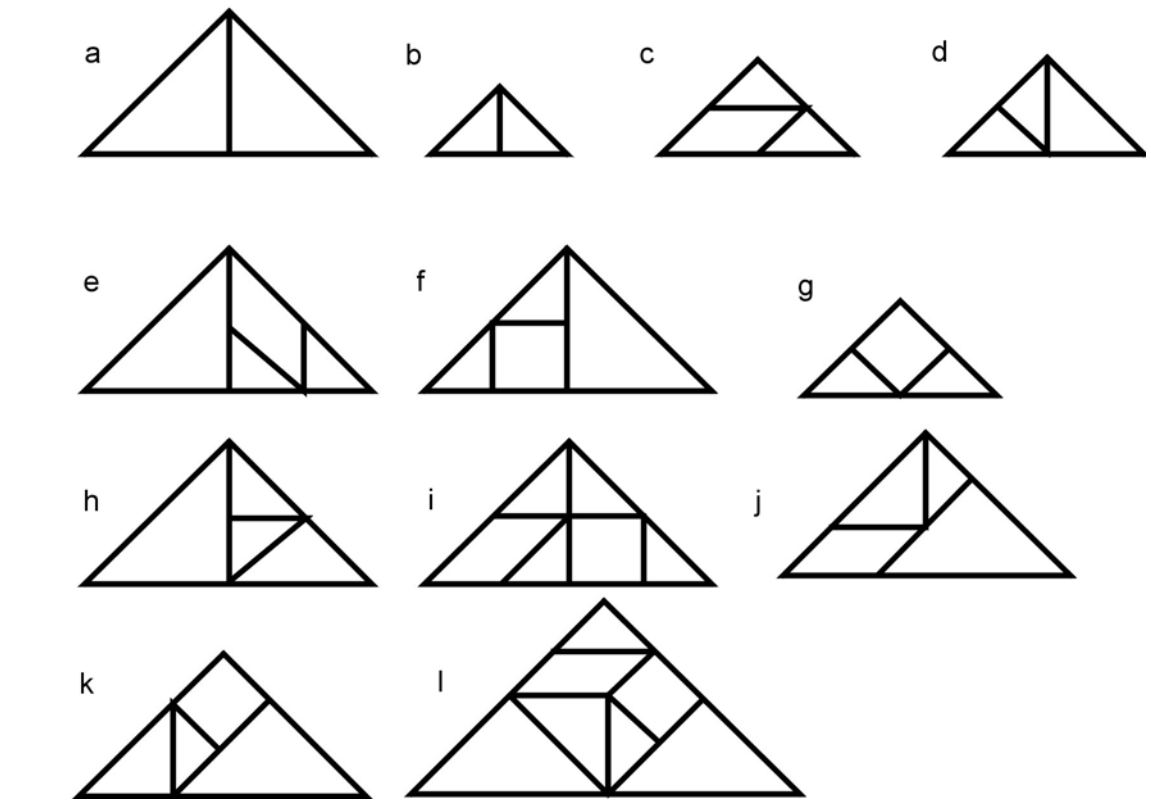
2.5: Squares and Right-Angled Isosceles Triangles

(Answers to BLM 2.3 and 2.4)

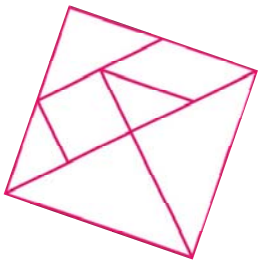
Squares:



Triangles



2.6: Shape Challenge Using Tangram Pieces



Name:
Date:

1. Using your set of tangram pieces, identify:
   - a square
   - a rectangle
   - a triangle
   - a parallelogram
   - a trapezoid

2. Use two tangram pieces to create as many of the shapes listed above as possible.

3. Use three tangram pieces to create as many of the shapes listed above as possible.

4. Use four, five, six, and seven pieces to create as many shapes as possible.

5. Put two or more tangram pieces together to form a shape that is congruent to another tangram piece.

6. Use the two small triangle pieces to make three different shapes.

7. Make a pentagon using two tangram pieces.

8. Make a hexagon using five tangram pieces.

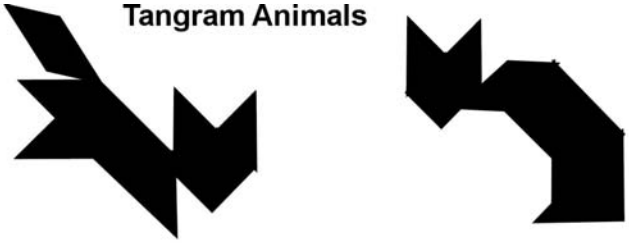
9. Use the five smallest tangram pieces to make a square.

10. Place the two large triangles around the square to form a triangle, a parallelogram, and a trapezoid.

2.7: Tangram Puzzles

Name:
Date:

Tangram Animals

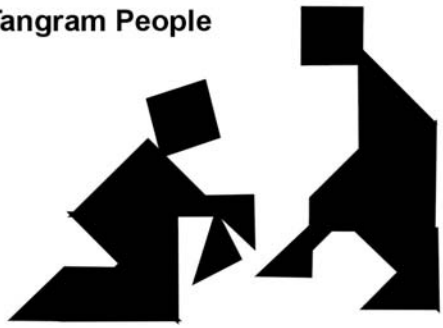


Fox

Cat

Chicken

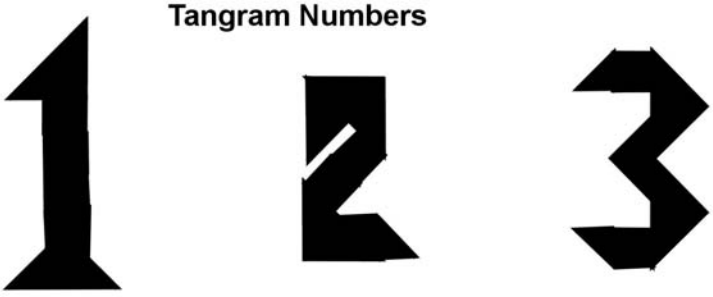
Tangram People



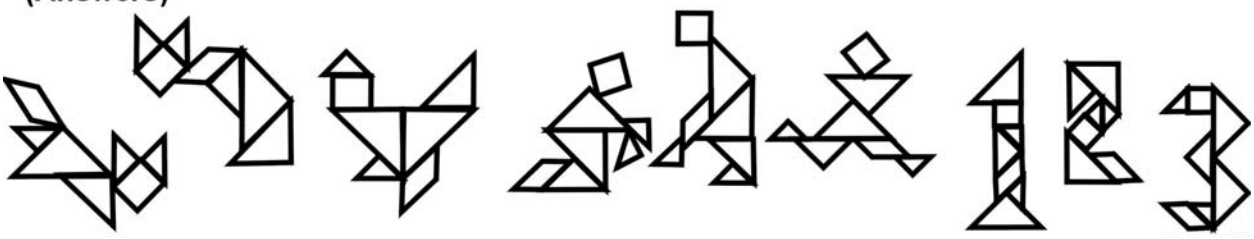
Shoeshine vendor and customer

Runner

Tangram Numbers



(Answers)



2.8: Area with Tangrams

Name:
Date:

1. Use your tangram pieces to complete the table. Consider the area of D to be one square unit.

Tangram piece	Calculated area of tangram piece	Fraction of the entire set (by area)
A		
B		
C		
D		
E		
F		
G		

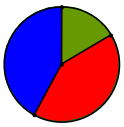
2. If the area of C is 2 cm^2, find the area of each of the other parts.

3. If the area of F is 3 cm^2, find the area of each of the other parts.

4. What fraction of part D is E?

5. What fraction of part A is C?

6. What fraction of part B is C?



Description
- Develop survey questions.
- Display data.

Materials
- graph paper
- graphing software

Assessment Opportunities

Minds On ...

Whole Class → Student Reports → Review

Ask several students to report on their tangram activities.

Recall kinds of graphs and their different purposes, e.g., circle graph shows parts of a whole; line graph shows change; bar graph shows relationship between separate items. Review important parts of a graph such as titles, labels, and scales.

Discuss how one "event" might prompt several questions for which different graphs might be appropriate, e.g., going to the beach.
- What type of graph would we use to show number of students likely to be at the beach at times during a day? (pictograph)
- What type of graph would we use to show the portion of the time spent at the beach swimming, playing volleyball, and/or sunbathing? (circle graph)
- What type of graph would we use to show to show favourite local beaches? (bar graph)

Small Groups → Brainstorm

Brainstorm seven topics that relate to summer activities. Form seven groups. Each group chooses one of the topics and prepares potential survey questions based on the topic. Discuss the importance of avoiding bias, asking a clear question, and having a fair sample in a survey. Each group shares its questions with the class. Questions are critiqued for suitability as part of a survey.

Action!

Pairs → Decision Making → Data Gathering → Graphing

Students formulate a question from the list discussed by the class, and predict the results.

Each pair writes their questions on separate pieces of paper with their names at the top. In a chain from student to student, allowing seven seconds, circulate these questions for the class to quickly respond to. A student near the end of the chain walks the paper to the other end of the chain. Stop the rotation of questions once enough data has been collected. Return papers to the pairs who posed the questions. Each pair chooses an appropriate graph type to display the data and prepares a graph to show the results.

Learning Skills/Observation/Rating Scale: Assess cooperation with others and class participation.

Note: Use a chain to avoid some students being left out or being unable to get the attention of their peers.

Consolidate Debrief

Whole Class → Sharing

Pairs show their graph and explain why the type of graph was chosen. Were the results as predicted? How would re-working the question change the graph?

Curriculum Expectations/Observation/Anecdotal: Assess appropriateness of the students' graphs.

Select a student to add vocabulary to the Word Wall.

Concept Practice

Home Activity or Further Classroom Consolidation

Present the data from your survey question in an alternative form, e.g., different scale, using technology.

Look for several types of graphs in print media and make a display.

Description

- Participate in a variety of tasks which may involve problem solving, patterning, performing calculations, working with 2-D shapes and 3-D models.

Materials

- dot paper
- cubes
- puzzles: word search, crossword
- playing cards
- number cubes
- BLM 4.1

Assessment Opportunities

Minds On ...

Whole Class → Reflection

Several students show their original and their revised graphs from the previous task, highlighting the effect of the changes.

Whole Class → Discussion

Brainstorm: What does "doing Mathematics" involve? Develop a concept map including problem solving, looking for or creating patterns, working with 2-D shapes or 3-D figures, and performing calculations.

Action!

Jigsaw → Activities

Prepare centres that might include:

- tangram puzzles not previously used;
- crossword puzzles, using geometric vocabulary;
- building seven-cube structures and recording them on dot paper;
- numerical skill improvement centre (game, not drill sheet);
- invitation to create a personal logo using geometric shapes;
- experiments
  (Example 1: When will a sum of 7 result if two number cubes are thrown? In 50 throws, how often will 7 occur? Predict the result in 1000 throws.
  Example 2: How many ways could someone draw two playing cards that would total 7?

Divide the class into home groups of 4-7 students, depending on the number of centres. Each home group sends a member to each centre for "expert training." At each centre, provide explicit written instructions so students can understand the activity. Experts return to their "home groups" and act as leaders when their group moves to the different centres.

Learning Skills/Observation/Checkbric: 4.1 Assessment Tool.

Consider choosing permanent centres based on strands or other criteria for which you might provide different activities.

Plan a schedule for home groups to move through several different centres in the room.

Select one centre and create a checklist of proficiency skills that can be observed.

Consolidate Debrief

Whole Class → Reflection

Students choose one activity and write about the mathematics that is used in it, referring back to the concept map created at the beginning of class. Selected students share their responses.

Home Activity or Further Classroom Consolidation

Make a math journal entry. How have you experienced success during these four days? Which were the most interesting activities? Which activities would you like to continue or do again?

Show parents or guardians the mathematics you completed. How do they use the skills of problem solving and communication in their daily lives (job, hobby, household)? What mathematical knowledge do they frequently use? Make notes on their comments to share with the class.

Reflection

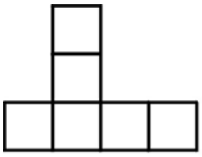
Respond to the student directly on the math journal page.

4.1 Assessment Tool

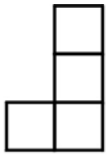
Select one of the activities. Create a checkbric (a combination of a checklist and rubric) to assess students' learning skills. A sample activity and corresponding checkbric are illustrated below.

Student Activity

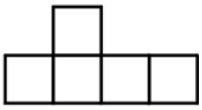
Use linking cubes to build a three-dimensional figure with the following views:



Front View

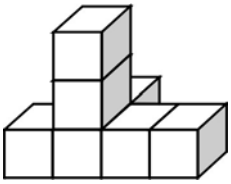


Side View



Top View

Solution:



Learning Skills	Needs Improvement	Satisfactory	Good	Excellent
Problem Solving				
• Applies logic in solving the problem				
• Perseveres in the task				
• Solves the problem independently				
Initiative				
• Responds to the challenge				
• Seeks assistance when necessary				

Lesson Outline: Days 5 - 9 Grade 7

BIG PICTURE

Students will:

- explore and generalize patterns;
- develop an understanding of variables;
- investigate and compare different representations of patterns.

Day	Lesson Title	Description	Expectations
5	Toothpick Patterns	• Review patterning concepts. • Build a growing pattern. • Explore multiple representations.	7m70, 7m72 CGE 3c, 4f
6	Patterns with Tiles	• Build a pattern. • Introduce the *n*th term.	7m66, 7m71 CGE 4b
7	Pattern Practice	• Continue development of patterning skills.	7m67, 7m71, 7m75 CGE 2c, 5e
8	Pattern Exchange	• Share work from previous day.	7m69, 7m75 CGE 2c, 5e
9	Performance Task	• Complete the performance task.	7m66, 7m67, 7m73, 7m75 CGE 5g

Description
- Review patterning concepts.
- Build a growing pattern.
- Explore multiple representations.

Materials
- toothpicks
- BLM 5.1

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Students contribute to a concept map about patterning, built on the board. Based on their experiences with patterning, they may identify types of patterns, materials for patterns, sample numerical or geometrical patterns, or applications of patterns.

Discuss why the ability to identify and discuss patterns is important. Ask a student to present a pattern on the board and another student to draw a different type of pattern. Have other students add the next term to each pattern and explain thinking. Use this as a sample to complete the understanding column in BLM 5.1. (See *Patterning to Algebraic Modelling* – Section 2)

Curriculum Expectations/Observation/Mental Note: Assess students' understanding of patterns, their confidence in using them, and their use of mathematical language.

Distinguish between a growing or diminishing "pattern" and a constant "design."

People use patterns to investigate and represent complex relationships existing in many areas, including nature and science.

Action!

Pairs → Activity

On an overhead, create the first two terms of the toothpick pattern presented on BLM 5.1. Ask a student to create the third term of the pattern.

In pairs, students continue the pattern with their toothpicks, and complete BLM 5.1. Circulate, offering assistance as required. Encourage students to look at different ways to build the "5th term, the "25th term," the "*n*th term," etc. There is no "right way" to formulate the construction of a term.

Students discuss solutions with their partners. Stress that each partner may have a different entry in the "understanding" column but should have the same value in the "number of toothpicks" column.

See TIP 1 – Multiple Representations – Pattern Building for examples of different ways students might see the pattern.

Consolidate Debrief

Whole Class → Discussion

Students share their approaches. Discuss different entries in the "understanding" column, highlighting the validity of all representations. Students should be using words and numbers but may not be using variables at this point.

It is important that students understand the limitation of recursive representations, e.g., add three to the last term. They should be encouraged to move to more sophisticated patterning, e.g., predicting the number of toothpicks required by consideration of the term number.

Each student should complete BLM. 5.1 during this class.

Concept Practice

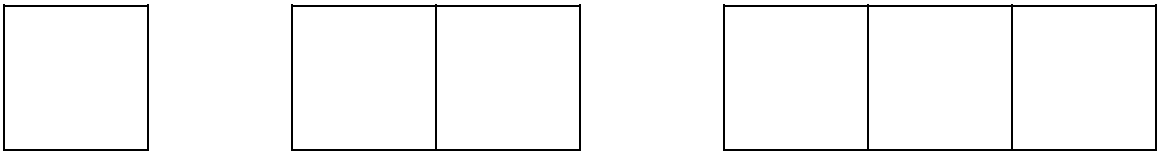
Home Activity or Further Classroom Consolidation

Design another toothpick pattern, building and recording the first three terms. Explain your pattern and consider how many toothpicks would be required to build the 100th term in the pattern.

5.1: Toothpick Patterns

Name:
Date:

1. Build this pattern with toothpicks.



Term 1 **Term 2** **Term 3**

2. Build the next two terms in the pattern.

3. Complete the chart. Put a numerical explanation of the number of toothpicks required in the Understanding column.

Term	Number of Toothpicks	Understanding
1		
2		
3		
4		
5		

4. How many toothpicks would you require to build the 100^{th} term? Explain your thinking.

5. Explain how to build the 100^{th} term another way.

Description

- Build a pattern.
- Introduce the *n*th term.

Materials

- colour tiles
- BLM 6.1

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Review concepts of patterns, and the idea that pattern building can be expressed in different ways. Selected students share patterns that they developed. Other students predict the number of toothpicks required for the 100^{th} term (BLM 5.1) and share their responses and strategies.

Encourage students to think about how they built each term in the pattern.

Curriculum Expectations/Observation/Mental Note: Collect diagnostic information during the discussion:

- which students are developing and using simplistic patterns
- which are developing more complex patterns
- which students are ready to move to variables.

Using a variable in reference to a term number contrasts use of a variable as a placeholder for a single unknown value in a linear equation.

Not all students will be ready for the former use of a variable in Grade 7, but all students should be comfortable with the latter use in Grade 7.

Action!

Pairs → Activity

Based on observations from the class discussion, pair students homogeneously according to their development level to allow for targeted assistance during the activity.

Student pairs complete BLM 6.1. Circulate, encouraging each partner to share her/his description.

Consolidate Debrief

Whole Class → Discussion

Ask a group to model on the board or overhead one representation for the tile pattern, writing their pattern clearly in words. Some students may choose to use variables. Encourage students to compare the various descriptions of the pattern, as well as the different representations (words versus algebraic) and discuss the advantages of each. Students should see that the various descriptions all represent the same situation and should look for connections between the descriptions.

Select a student to add vocabulary to the Word Wall - *term, term number, variable.*

Home Activity or Further Classroom Consolidation

Revisit your toothpick patterns. Find two other ways to express your pattern. Consider other rules for generating the same pattern and/or express the pattern using variables, if appropriate.

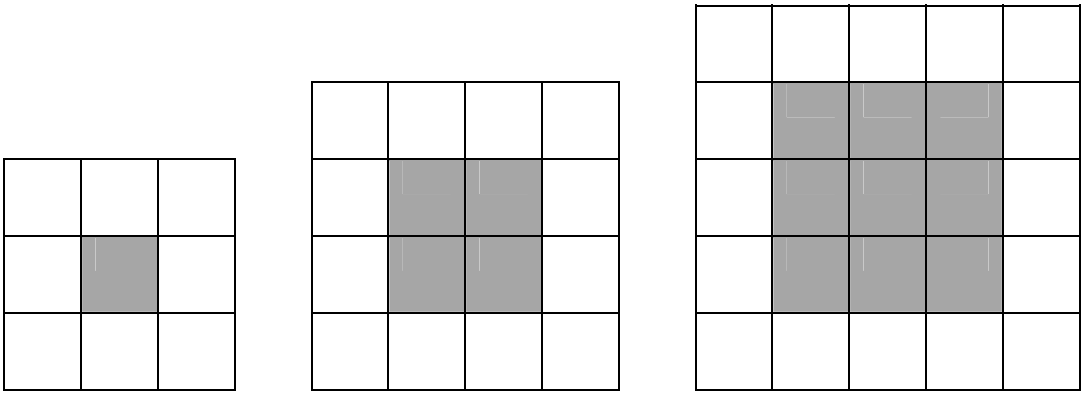
Discuss the use of patterns with an adult. In your math journal, describe how they use patterns in their career or hobbies.

Concept Practice
Exploration
Skill Drill

6.1: Patterns with Tiles

Name:
Date:

1. Build the first five terms of this sequence using light and dark tiles.



2. Complete the table.

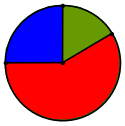
Term Number	Number of Light Tiles	Understanding	Number of Dark Tiles	Understanding
1				
2				
3				
4				
5				

3. a) How many dark tiles are there in the 10^{th} term? Explain your reasoning.

   b) How many light tiles are there in the 10^{th} term? Explain your reasoning.

4. How many light tiles are there in the 100^{th} term? Explain your reasoning.

5. Describe a strategy for working out how many dark tiles and how many light tiles are needed to build any term.



Description
- Continued development of patterning skills.

Materials
- manipulatives, e.g., tiles, toothpicks
- BLM 7.1, 7.2
- Developing Mathematical Processes questions

Assessment Opportunities

Minds On ...

Groups of 4 → Placemat

To heighten their awareness of linkages between mathematics and life experiences, students share the ideas they determined about using patterns in jobs and hobbies.

Action!

Small Group → Activity

Curriculum Expectations/Exhibition/Checklist: Circulate and observe students as they complete BLM 7.1. Assess their ability to recognize a pattern, extend a pattern, express the pattern using words and/or algebraically, and determine a value for a non-consecutive term (such as 100th term).

Students work in groups of three to complete BLM 7.1. Provide assistance, as required.

On chart paper or on the board, students record their responses to b) and c) for each of the three patterns on BLM 7.1. Allow a portion of the class for students to add their method for describing the 20th and the *n*th terms to the charts/board. Students should put their initials beside their response. Students create their own patterns (BLM 7.2).

Some students may move to abstract representations, while others may continue to use concrete materials.

Refer to TIP 1 – Multiple Representations – Pattern Building

Consolidate Debrief

Pairs → Sharing

Students read over the responses from one section of BLM 7.1, such as 1b. Allow for some individual interpretation. Facilitate a dialogue between the students, encouraging them to ask clarifying questions of each other.

Note: Students may respond using a variety of representations (words and/or algebraic expressions). Students' descriptions of their pattern and their representations may vary; however their representations should be equivalent.

Encourage students to identify equivalent expressions that look significantly different and explain how they determined equivalency.

Students could discuss the equivalency of different representations and expressions, e.g., different but equivalent representations may be "double the number" or "add the number to itself" or "multiply the number by two" or "2*n*."

Concept Practice

Home Activity or Further Classroom Consolidation

Complete worksheet 7.2. Have an adult answer your three questions. Pay close attention to the process that they use to answer these questions. Record their process in your math journal and identify if it is the same or different from your process.

Complete the questions, extend a pattern, complete a table, and write words to explain the pattern.

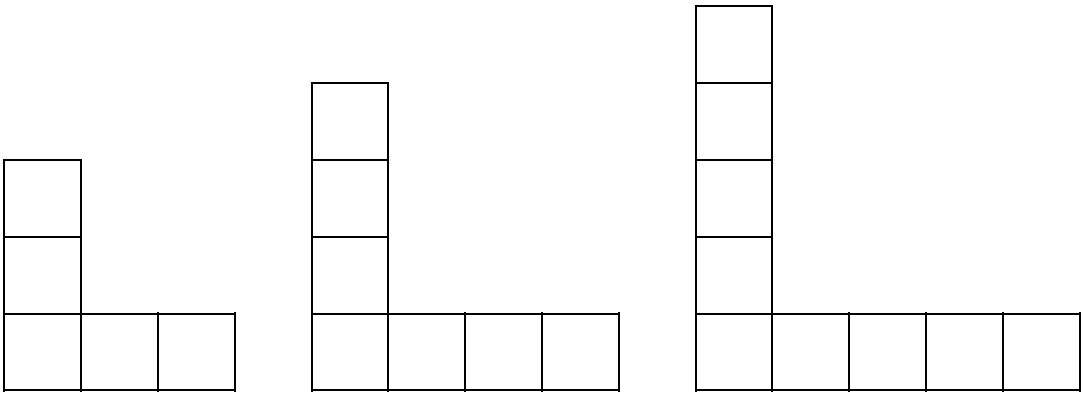
See *Patterning to Algebraic Expressions* – Section 2 *Developing Mathematical Processes* p. 5.

7.1: Pattern Practice

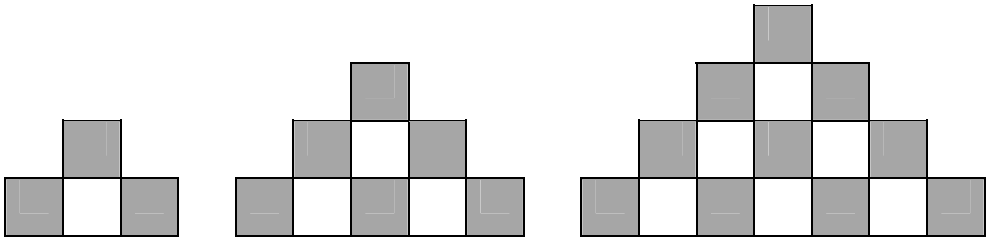
Name:
Date:

For each example below:
a) build the first few terms of the pattern
b) write at least two different ways to describe how to build the 20^{th} term
c) write at least two different ways to describe how to build the *n*th term.

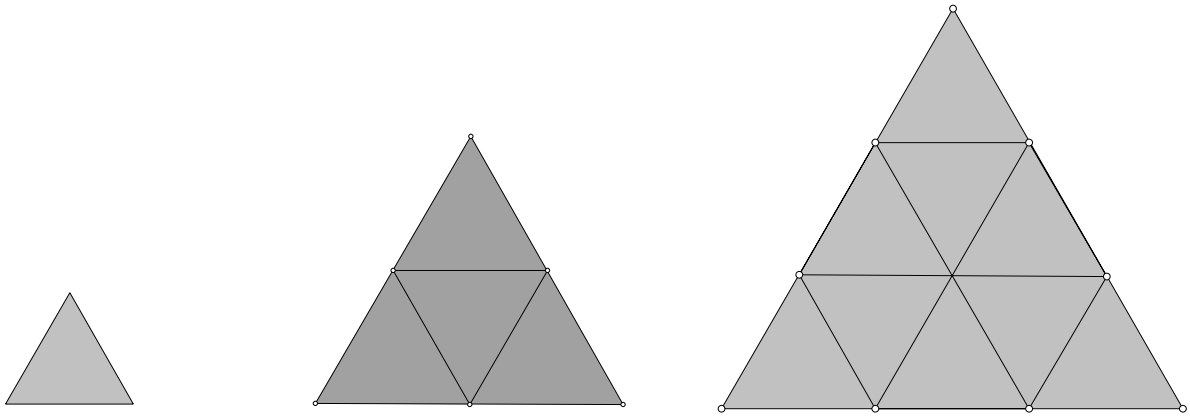
1.



2.



3.



7.2: Pattern Posing

Name:
Date:

Create your own pattern using tiles, toothpicks, or another material.

Materials used: _____

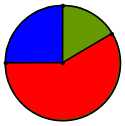
A drawing of the first three terms of my pattern

Three questions that someone could answer about my pattern are:

1.

2.

3.



Description
- Share work from previous day.

Materials
- completed BLM 7.2
- colour tiles, toothpicks

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Students share their solutions to the Developing Mathematical Processes questions from Day 1. Validate various correct representations. Select some expressions that are equivalent and have students justify that they are. It is important to value variety in responses so that the stage will be set for algebraic manipulations introduced when one representation is more appropriate for particular applications. Students recognize that some representations should be based on the term number, not the value of the previous term (functional, not recursive).

Curriculum Expectations/Observation/Anecdotal: Make a note of students who are still defining patterns recursively in order to provide some direct assistance during the activity.

By generating multiple responses for one question, the teacher shows that thinking is valued, as well as correct answers.

Action!

Individual → Activity

Use a chain to re-distribute students completed BLM 7.2. (See Differentiated Instruction below.) Students record their responses on a paper, with the creator's name and their name at the top. Use a chain to re-distribute questions again so that students respond to the patterns created by two different students.

Curriculum Expectations/Self-Check/Anecdotal: Each student should receive their BLM 7.2 and two responses from their peers. They review their peers' work and discuss their solutions as required.

Students should go to the author for clarification, rather than the teacher.

Differentiated Instruction

The sharing of BLM 7.2 could happen within three groups in the classroom. The first group could be the students who are comfortable creating general terms and may use algebraic representation. The second group could be the students who encounter slight difficulty with the concept but are usually able to express a general term, perhaps using words. The third group would be the students identified by the teacher in the Minds On section. The teacher may choose to work directly with the third group on the process for generating a rule based on the term number.

Consolidate Debrief

Whole Class → Discussion

Students share interesting patterns that they encountered during the lesson and some of the dialogue that occurred during the self-check process.

A general debriefing of the activity allows the teacher to see where the students are in their understanding of patterns.

Home Activity or Further Classroom Consolidation

Reflection
Concept Practice

In your math journal, reflect on your individual skill development. Can you:
- extend a pattern?
- describe a pattern in words?
- use a pattern to make a prediction?
- determine a specific term (such as the 100^{th} term) by referencing the term number rather than the previous term?
- use appropriate language to describe the pattern?

Complete the following questions for additional practice: (The teacher inserts text references.)

Description
- Complete the performance task.

Materials
- BLM 9.1
- colour tiles

Assessment Opportunities

Minds On ...

Whole Class → Set the Stage

Explain to students that they will be working individually on an activity that is similar to the ones completed during the past few lessons. Discuss with students the rubric on which they will be assessed.

Students read over BLM 9.1 and ask any clarifying questions.

Action!

Individual → Assessment Activity

Students complete BLM 9.1

Assign extending or enriching tasks before the students begin so that those students who finish early can quietly move on to these extensions. Those who do not have time to work on the extensions can do them outside of class.

Circulate as students work. If you notice a student who is not making progress, consider:

- giving the student manipulatives to build models of pools and patios;
- observing the student building or drawing the 4th diagram and ask prompting questions.

If a student is generating a low level response, ask prompting questions on the student's answer sheet, using colourful ink. The student responds in writing. This written record of prompting questions and answers can be referred to during a student or parent interview.

Curriculum Expectations/Written Work/Rubric: Assess students demonstration of their learning, using a rubric.

For samples of student work refer to The Ontario Curriculum, exemplars, Grade 7, Mathematics.

See the sample rubric in the exemplar.

Consolidate Debrief

Whole Class → Set Required Proficiency Level

Collect completed student work. Distribute copies of a sample Developing Proficiency test based on the curriculum expectation: extend a pattern, complete a table, and write words to explain the pattern. See Section 2 – *Patterning to Algebraic Modelling* content-based package. Explain that all students need to develop proficiency on these sorts of questions and that each student will have the opportunity to try a similar test and get help if needed until they meet the required proficiency level.

After students have had a chance to complete a sample Developing Proficiency test independently, take up solutions and answer questions.

Curriculum Expectations/Proficiency Test/Marking Scheme: When it appears that most students in the class are ready to demonstrate their proficiency, give students another version of the test to do independently. Collect and score checking whether the proficiency target has been met or proficiency is still developing. When returning scored tests, provide another teaching loop for students who did not meet the target while those who met the target work on extensions.

When returning graded work to students, consider photocopying samples of Level 3 and 4 responses with student names removed. Select and discuss, with the class, samples that show a variety of strategies.

Home Activity or Further Classroom Consolidation

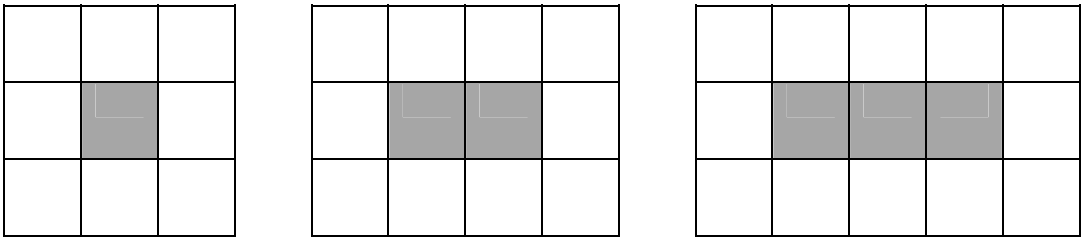
Exploration
Skill Drill

Pick a number between 5 and 10. Complete the multiplication facts for that number from 1 to 12. Identify patterns in the multiplication facts.

9.1: Performance Task

Name:
Date:

1. A rectangular pool is surrounded by a patio. Below are the first three terms of a pattern for designing the pool. The dark squares represent the water in the rectangular pool and the light squares represent the patio.



   a) If the pool has an area of twenty-five square units, how many tiles are needed for the patio? Explain your thinking.

   b) If the pool has an area of *n* square units, how many tiles are needed for the patio? Show how you arrived at your answer.

   c) If there are 206 patio tiles, what size of a rectangular pool can you build? Explain your thinking.

2. a) Use two different coloured tiles and construct a different arrangement for a pool and a patio.

   b) Describe how you would find out how many tiles would be in the thirteenth pool.

Source: The Ontario Curriculum, Exemplars, Grade 7, Mathematics, 2002.

Lesson Outline: Days 10 - 12 Grade 7

BIG PICTURE

Students will:

- recognize the importance of problem solving and investigation in learning mathematics;
- develop an understanding of factors and multiples and of the relationship between them;
- generate factors and multiples of given numbers.

Day	Lesson Title	Description	Expectations
10	Precious Conjectures	• Explore some conjectures about powers and primes. • Examine factors, multiples and products of prime numbers, square numbers, and odd numbers as they prove or disprove a variety of conjectures.	7m11, 7m67, 7m71 CGE 3c, 4b
11	Investigating Patterns in Factors	• Identify factors for a range of numbers. • Analyse the information and complete questions based on the information.	7m11 CGE 5d, 5g
12	Venn Diagrams	• Use Venn diagrams to classify factors of two and three numbers. The stage is set for common factors located in the intersection of regions to be used in Term 2 when addition of fractions is studied. Venn diagrams for multiples are included. • Create their own diagrams and answer some in-depth questions.	7m11, 7m25 CGE 2a, 2c

Description

- Explore some conjectures about powers and primes.
- Examine factors, multiples and products of prime numbers, square numbers, and odd numbers as they prove or disprove a variety of conjectures.

Materials

- BLM 10.1, 10.2

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Describe how some mathematicians spend years attempting to prove or disprove a conjecture or theory. Sometimes practical applications of these theories are not evident for years afterward. Students may recognize Einstein's Theory of Relativity, $E = MC^2$ as an example of a conjecture.

Lead a discussion on Goldbach's Conjecture: "Every even number greater than 2 can be written as the sum of two prime numbers." Students provide numeric examples to prove or disprove the conjecture as they answer the three questions on BLM 10.1.

Examples to support Goldbach's Conjecture:

$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 = 3 + 7$

Students work in pairs to prove or disprove further conjectures. To do so, they must understand certain terminology.

Bronzebach's Conjecture requires knowing that perfect squares are 1, 4, 9, 16, 25, 36, 49, … and that these can be written as 1^2, 2^2, 3^2, 4^2, 5^2, …
Tinbach's Conjecture refers to the prime numbers 2, 3, 5, 7, 11, 13, 17…
Brassbach's Conjecture refers to factors or divisions of a number.

Ensure that students understand that every number has 1 and itself as factors.

A conjecture is a statement that may appear to be true, but has not yet been proven.

In honour of Goldbach, mathematicians humorously named several other prime number conjectures after metals and minerals. (Tinbach's, Copperbach's, Aluminumbach's Conjecture)
Source: Helping Children Learn Mathematics, 6th ed., Wiley Europe, 2002.

A prime number has only 2 different factors: 1 and itself.

Action!

Pairs → Activity

Curriculum Expectations/Observation/Mental Note: Observe students' mental arithmetic skills, appropriate use of calculators, and problem solving skills as they work on the activity.

Working in pairs or groups of three, students complete BLM 10.2 to prove or disprove each of the remaining conjectures. Students complete each of the three questions for each conjecture, as modelled for Goldbach's Conjecture.

"There are an infinite number of prime numbers" is a conjecture that many mathematicians have worked on.

Consolidate Debrief

Whole Class → Debrief

Students present their findings. Lead a discussion about what would be enough to prove a conjecture true or to disprove or refute a conjecture. Most students will agree that you can prove a conjecture if you show all possible cases to be true. Many students will not think it possible to establish a proof if there are an infinite number of cases. A few students may be interested in researching "proof by mathematical induction."
The following counter-example is sufficient to disprove or refute Brassbach's Conjecture: 16 has 5 factors 1, 2, 4, 8, 16, not just 3 factors.

Prime numbers are of particular interest to those who create security codes.

Select a student to add vocabulary to the Word Wall: *power, prime, product*, and *factor.*

Concept Practice

Home Activity or Further Classroom Consolidation

Complete worksheet 10.2 for Precious Conjectures about Powers and Primes.

10.1: Goldbach's Conjecture

Name:
Date:

"Every even number greater than 2 can be written as the sum of two prime numbers."

$4 = 2 + 2$

1. Verify that the conjecture is true for the first ten numbers in the pattern.

2. Select five additional numbers (not necessarily consecutive or immediately following the first 10) and verify that the conjecture is also true for them.

3. Suppose that you were able to write valid statements for the first 1 000 000 numbers in the pattern. Would this be enough to prove that the conjecture is true? Explain your reasoning.

10.2: Precious Conjectures about Powers and Primes

Name:
Date:

For each of the conjectures listed:

- verify that the conjecture is true for the first ten numbers in the pattern.
- select five additional numbers and verify that the conjecture is true for them. These numbers do not have to be consecutive or immediately follow the first ten numbers.
- suppose that you were able to write valid statements for the first 1 000 000 numbers in the pattern. Would this be enough to prove that the conjecture is true? Explain your reasoning.

Conjectures

Bronzebach's Conjecture

Every natural number is the sum of four or fewer perfect squares.
$10 = 3^2 + 1^2$

Tinbach's Conjecture

Every number can be expressed as a difference of two prime numbers.
$14 = 17 - 3$

Copperbach's Conjecture

The product of any number of prime numbers is odd.
$5 \times 3 \times 7 = 105$

Aluminumbach's Conjecture

Every odd number can be expressed as the sum of three primes.
$11 = 3 + 3 + 5$

Challenge

Brassbach's Conjecture states that "Every square number has exactly 3 factors." One example is 2^2, with factors 1, 2, and 4. Investigate this conjecture using the first 10 perfect squares. Write a brief explanation of your results and explain how they confirm or refute the conjecture.

Description

- Identify factors for a range of numbers.
- Analyse the information and complete questions based on the information.

Materials

- BLM 11.1, 11.2
- graph paper
- calculators

Assessment Opportunities

Minds On ...

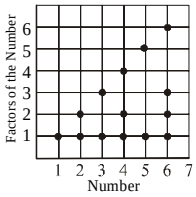
Whole Class → Teacher Directed

Students connect their prior knowledge of factor, prime and multiple by doing the following mental math problems. As students work through the identification of the factors, they share strategies for determining factors (even numbers divisible by 2, rules for divisibility by 9, etc.). Identify with them organization strategies to ensure that no factors are omitted.

- A number is ***perfect*** if all its factors, other than the number itself, add up to the number.
  Students prove that 28 is a perfect number. Students determine the only one-digit perfect number. (6)
- A number is ***abundant*** if the sum of all its factors, other than the number itself, is greater than the number. A number is ***deficient*** if the sum of all its factors, other than the number itself, is less than the number.
  Are 12 and 5 abundant or deficient numbers?

Pose the question: Are there patterns found in the factors of the numbers from 1-50?

Model the investigation by determining the factors of the first 6 numbers and placing them on an overhead grid (BLM 11.1).



A number is divisible by 9 if the sum of its digits is divisible by 9.

Factors of 28 are 1, 2, 4, 7, 14, and 28.
28 = 1+2+4+7+14
Thus, 28 is a perfect number.

Factors of 12 are 1, 2, 3, 4, 6, 12.
1 + 2 + 3 + 4 + 6 > 12, thus 12 is an abundant number.

Numbers that have 2 as a factor are called "even" numbers or multiples of 2.

Action!

Small Groups → Activity

Curriculum Expectations/Observation/Mental Note: Gather assessment information about individual student's number sense by observing while they complete the investigation.

In groups of three or four, students continue the investigation for the numbers 7-50 breaking the task into smaller parts. Allow them to use calculators. They record the results on graph paper for all numbers 1 through 50, so that patterns are visible. Once they complete their grid, the groups answer the questions on BLM 11.2, noting patterns that occur.

Some students may use calculators extensively, while others will use their knowledge of number facts and patterns to complete the factor grid.

Consolidate Debrief

Whole Class → Clarification/Wrap-Up

Students share their solutions to the Factor Questions (BLM 11.2) with the class.

To facilitate class discussion, use a completed overhead grid that illustrates the solution.

Skill Drill

Home Activity or Further Classroom Consolidation

For the numbers from 1 to 20, determine which numbers are *abundant*, *perfect*, and *deficient*.

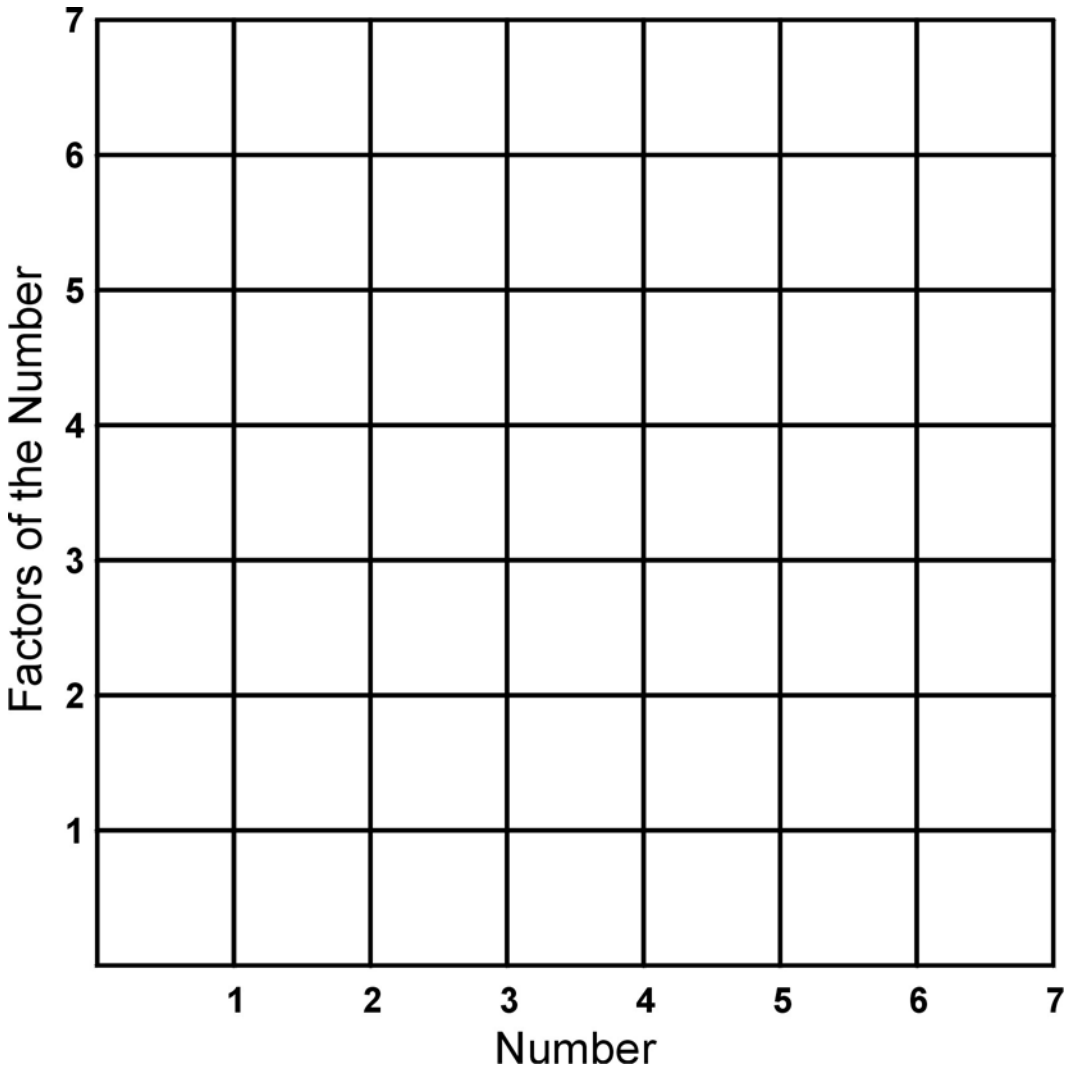
11.1: Let's Investigate

Name:
Date:

Are patterns created by the factors of the numbers from 1-50?

Above each number along the horizontal axis, place a dot opposite each of its factors.
Example: The factors of 6 are 1, 2, 3, and 6.

Complete the factor grid for numbers 1-7.



11.2: Factor Questions

Name:
Date:

1. What observations and patterns do you notice on the factor grid (worksheet 11.1)?

2. List the numbers for which 5 is a factor.

   How would you describe these numbers?

3. In two different ways, describe the numbers that have 2 as a factor.

4. Identify the numbers that have only two factors.

   What is the name given to these numbers?

5. a) From the factor grid, list:
   - numbers that have both 3 and 6 as factors.

   - numbers that have both 3 and 9 as factors.

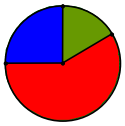
   b) Compare the two lists. What patterns do you notice about multiples of 6 and multiples of 9?

   c) How might these patterns help a student who is trying to learn the multiplication facts for 6 and 9?

6. On the factor grid, examine the column of factors for 12 and 18.
   a) Where on the factor grid do you find the greatest common factor of 12 and 18?

   b) Use the factor grid to determine the greatest common factor of 24 and 42.

7. Consider the number 120. How might you use the information in your factor grid to determine all the factors of 120?



Description

- Use Venn diagrams to classify factors of two and three numbers. The stage is set for common factors located in the intersection of regions to be used in Term 2 when addition of fractions is studied. Venn diagrams for multiples are included.
- Create their own diagrams and answer some in-depth questions.

Materials

- BLM 12.1

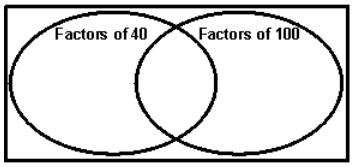
Assessment Opportunities

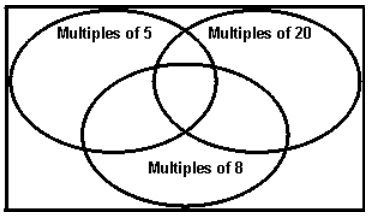
Minds On ...

Whole Class → Teacher Directed

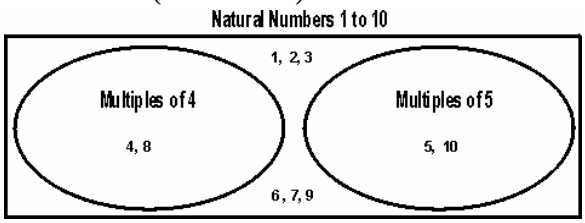
Students share their solutions for *abundant*, *deficient* and *perfect* number work.

Model two Venn diagrams on the board.





Give students a few minutes to try the questions. Students orally provide numbers for each region of the Venn diagram. Recall that numbers that are not factors of either (such as 7) should be listed outside the circles but inside the box. The box (universal set) contains all of the numbers students must consider.



If there are no common elements, ovals in the Venn diagram may not overlap.

Action!

Pairs → Thinking Activity

Students may need reminders about how to record the information correctly in a Venn diagram. Consider only numbers from 1 to 100.

Students work in pairs to complete BLM 12.1. Encourage them to identify ways to self-check their work, using patterns and number sense.

Groups of 4 → Checking

Each pair joins with another pair to form a group of four to compare answers and resolve any discrepancies.

Each group of four creates two more Venn diagrams to challenge classmates. They include solutions separately.

Pairs → Thinking Activity

In pairs, students work through the Venn diagrams created by their peers.

Consolidate Debrief

Whole Class → Discussion

Curriculum Expectations/Quiz/Marking Scheme: Give a short quiz on factors, multiples and primes.

Students state their observations and understandings about factors and multiples. This may include comments such as: all even numbers have 2 as a factor; if 6 is a factor then 3 and 2 are factors, all multiples of 9 have digits that have a sum of 9.

Discuss and validate these observations and understandings.

Home Activity or Further Classroom Consolidation

In your math journal, write a summary of the rules, understandings, and concepts that you learned about factors.

Complete the following questions for additional practice: (The teacher inserts text references.)

Reflection
Skill Drill

Students' oral answers provide a diagnostic assessment opportunity.

The natural numbers for 1 to 100 would be an appropriate choice for these questions.

Pairing students by ability allows those with a strong understanding to move quickly to more challenging tasks.

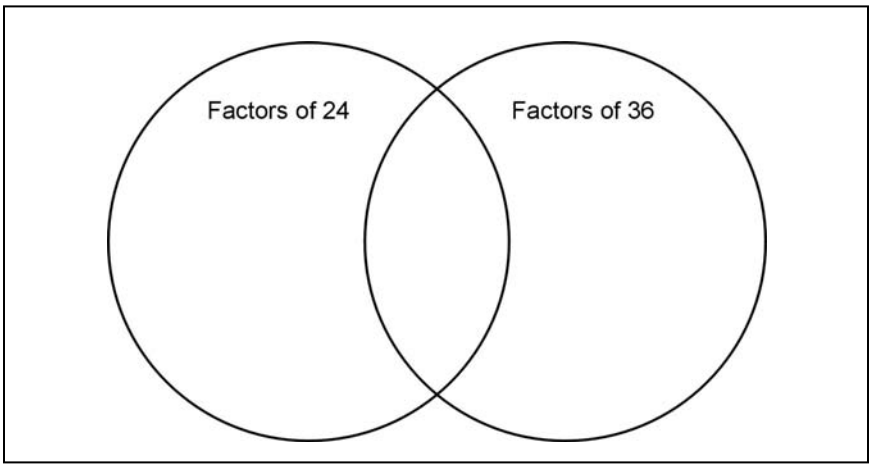
Note: When checking for divisibility by 9, if the sum is not a one-digit number, continue adding the digits until a one-digit number occurs. e.g., $9 \times 13 = 117$, add the digits of the multiple 117 to get $1 + 1 + 7 = 9$

12.1: Venn Diagrams

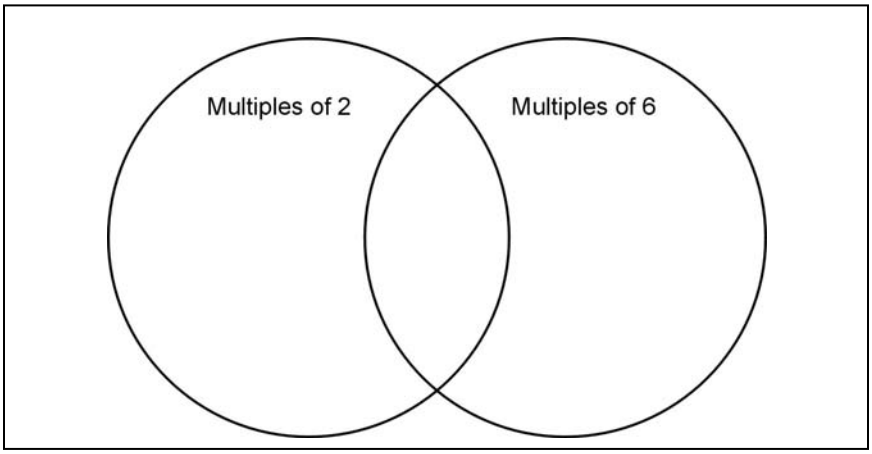
Name:
Date:

1. Complete the following Venn diagrams using Natural Numbers 1 to 100.

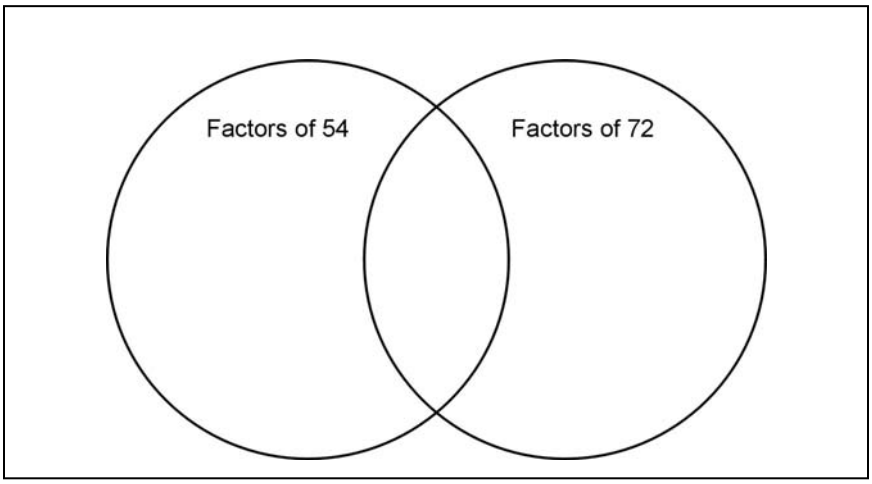
a)



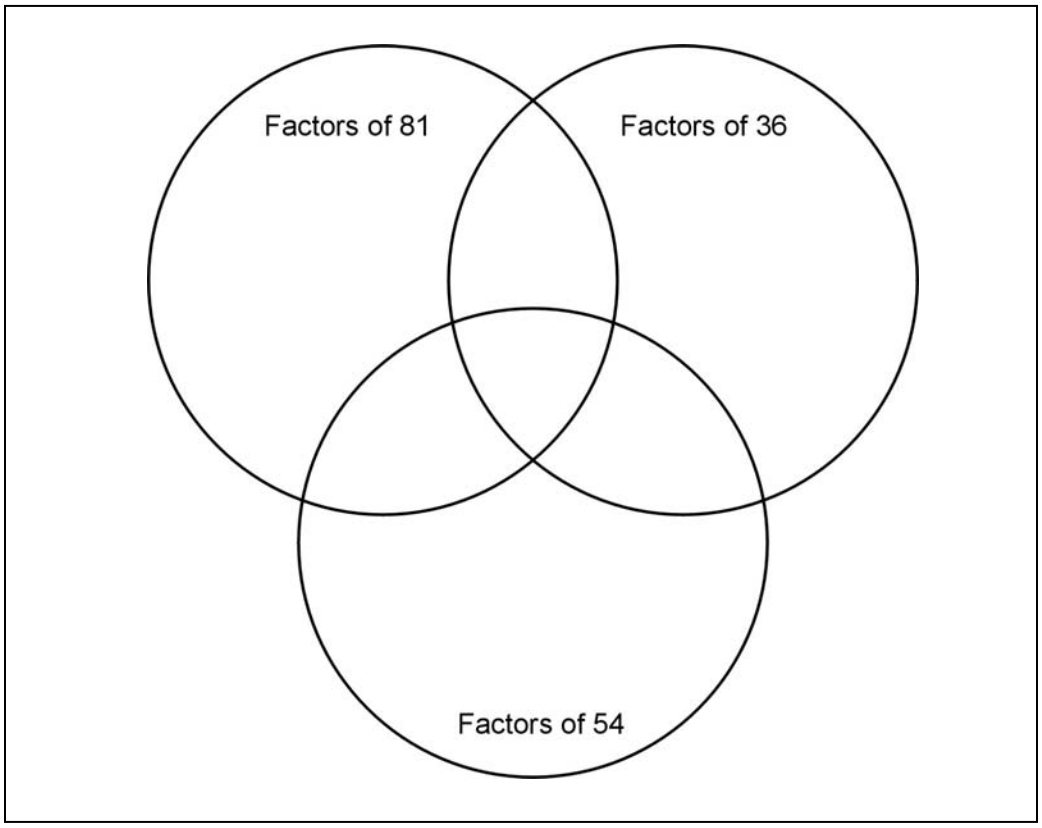
b)



c)



d)



2. Identify pairs of numbers between 1 and 50 that would create the following situations in a Venn diagram:
   a) Three numbers that would have only one factor in common.

   b) Three numbers that would have many common factors.

   c) Two numbers that would not have any multiples in common.

   d) Three numbers that would not have any common multiples.

   e) Three numbers that would have many common multiples.

3. Is it possible to select a pair of numbers that would have no common factors? Explain your thinking.

4. What number has the largest number of multiples in between 1 and 50? Show your work.

5. What number less than 50 has the least number of factors? Show your work.

Lesson Outline Days 13 - 16 Grade 7

BIG PICTURE

Students will:

- relate areas of parallelograms, rectangles, squares, rhombi, and triangles;
- develop inquiry skills;
- develop strategies for estimating and calculating the area of polygons.

Day	Lesson Title	Description	Expectations
13	Area of a Parallelogram	• Develop and apply a formula for the area of a parallelogram. • Understand the relationship between base and height of a 2-D shape.	7m25, 7m5, 7m28, 7m33, 7m35, 7m41, 7m13, 7m29 CGE 2a, 4b
14	Area of a Triangle	• Develop and apply the formula for finding the area of a triangle.	7m25, 7m28, 7m35, 7m41 CGE 2a, 3c
15	Area of Polygons	• Use formulas and diagrams to find the area and perimeter of irregular polygons (composite figures).	7m16, 7m47, 7m24, 7m25, 7m33, 7m30 CGE 2c, 3c
16	Cumulative Assessment	• Develop a formula for finding the area of a trapezoid.	7m35 CGE 5g

Day 13: Area of a Parallelogram — Grade 7

Description

- Develop and apply a formula for the area of a parallelogram.
- Understand the relationship between base and height of a 2-D shape.

Materials

- grid paper
- geoboards
- *The Geometer's Sketchpad®*
- BLM 13.1

Assessment Opportunities

Minds On ...

Whole Class → Brainstorm

On the board, create a list (with diagrams) of different kinds of triangles and quadrilaterals. Discuss the characteristics of each, using questions: What are the distinguishing features of each kind of triangle? Is every square a rectangle? Is every rectangle a parallelogram? Is every parallelogram a rectangle? Are the words "side" and "height" interchangeable?

What's the difference between area and perimeter? Outline the perimeter and shade the area for several shapes, using colour to help visual learners.

The dynamic parallelogram and rectangle in *The Geometer's Sketchpad®* demo provide powerful visual confirmation and counter-examples of relationships between and among shapes.

Action!

Pairs → Teacher Guided

Using geoboards, students create a variety of rectangles, orally calculate their areas and perimeters and explain how they selected appropriate units and distinguish between the two measures. Show their responses on an overhead.

Using grid paper, students create a rectangle, cut it out, and calculate the area, using colour to mark the measurements. They draw a line from a vertex to a point on the opposite side and cut along the line. They slide one piece over the other until sides that used to be opposite are on top of each other. Ask: What shape has been formed? Has the total area changed? Where are the original measurements?

See the content-based package – *Developing Perimeter and Area Formulas*.

Discuss how to determine where the base and height are in the new shape.

Think/Pair/Share → Practice

On grid paper, students create five very different parallelograms. They trade with a partner who uses colour to mark the base and the height and then calculate the area of each. Students exchange papers and check their partner's work.

Stress that the base and the height of any polygon are at right angles to each other.

Whole Class → Teacher Guided

Curriculum Expectations/Question & Answer/Mental Note: Using an overhead of BLM 13.1, check for understanding. Uncover a diagram, ask students to make mental decisions about base and height, and after sufficient wait time, draw in the base and height in colour. Ask students to mentally calculate, then write the final area calculation. Provide oral feedback as students answer questions. Use the last few examples to demonstrate acceptable form in written solution.

Make transparency cut-outs of the shapes on BLM 13.1.

Consolidate Debrief

Whole Class → Notemaking

Construct a Venn diagram together as shown on Answer sheet 13.1.

Orient parallelograms, rectangles, squares, and rhombi in various ways on an overhead transparency and invite students to identify a base and height for each shape. (There will be two base-height pairs for each shape.)

Draw a visual reminder of the equality of areas of parallelograms and rectangles on the same base and between the same parallel lines.

Select a student to add vocabulary to the Word Wall.

Home Extension or Further Classroom Consolidation

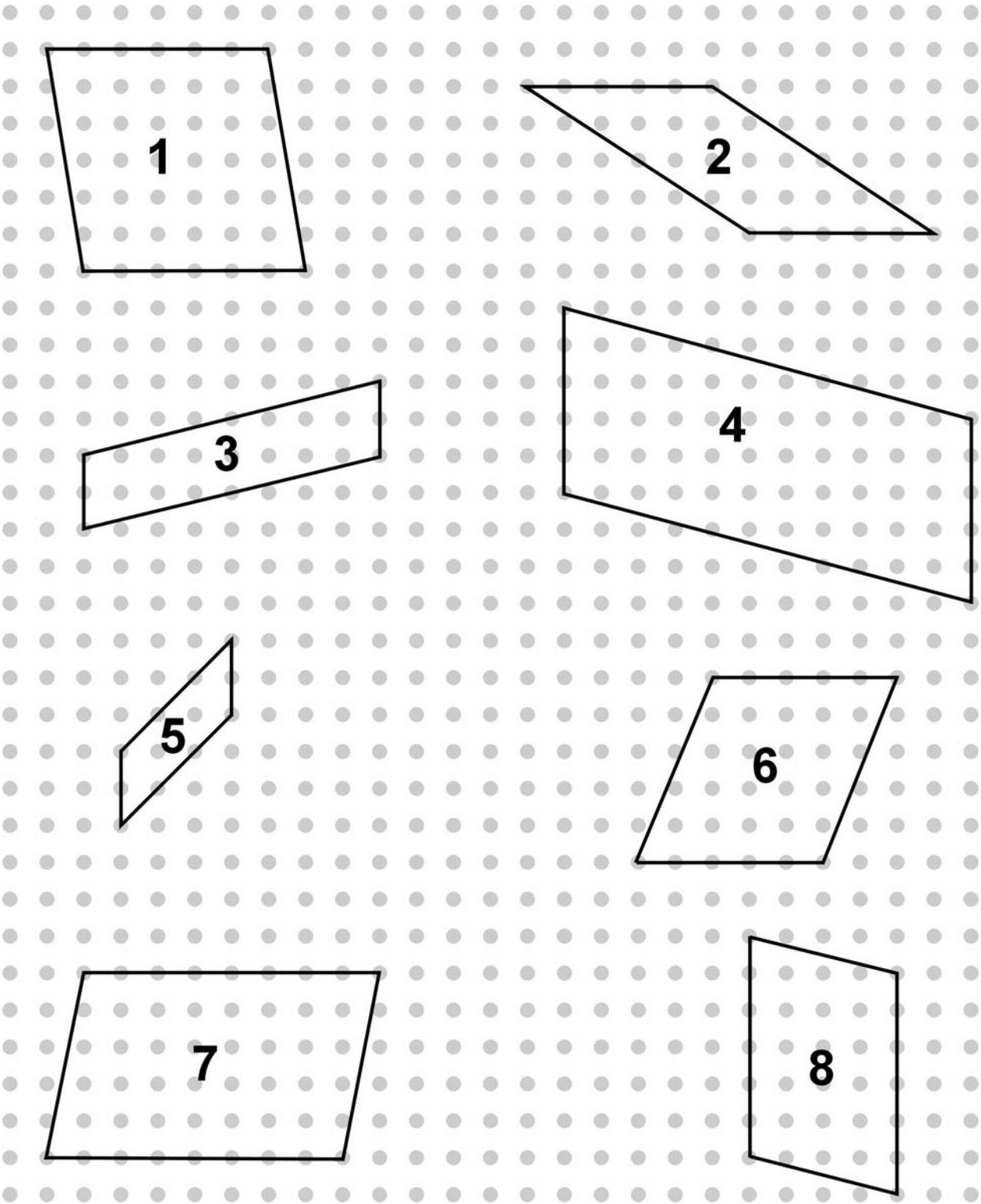
Identify different quadrilaterals found in your home. Take necessary measurements; draw a sketch for each one; and calculate its area.

Exploration
Skill Drill

In your math journal, answer the questions and give examples to justify your response: Can quadrilaterals with the same perimeter have different areas? Can quadrilaterals with the same area have different perimeters?

13.1: Parallelograms

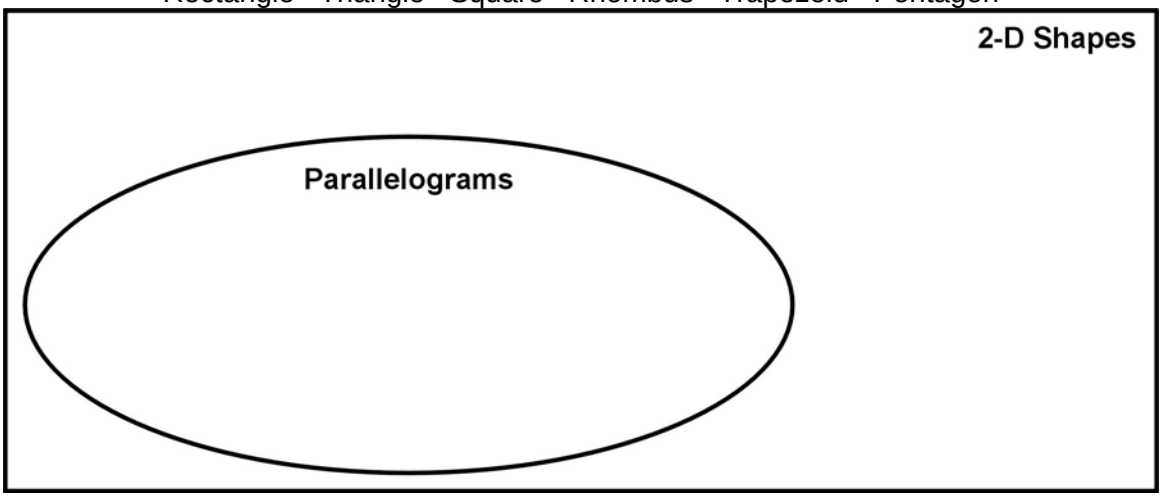
Name:
Date:



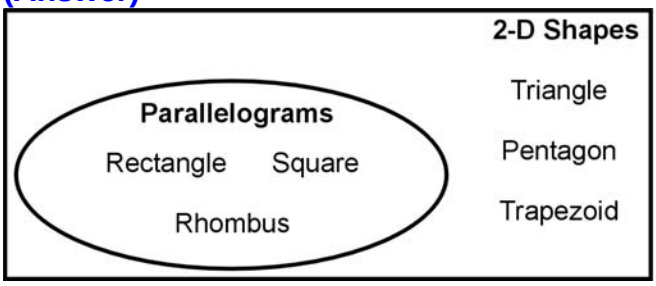
13.1 Two-Dimensional Shapes Answer Sheet (continued)

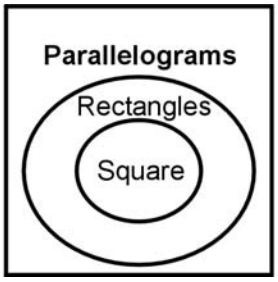
(Question)

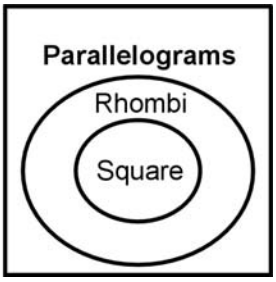
Rectangle Triangle Square Rhombus Trapezoid Pentagon



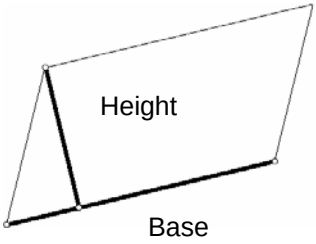
(Answer)

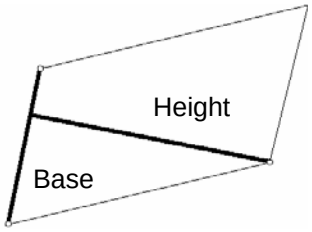




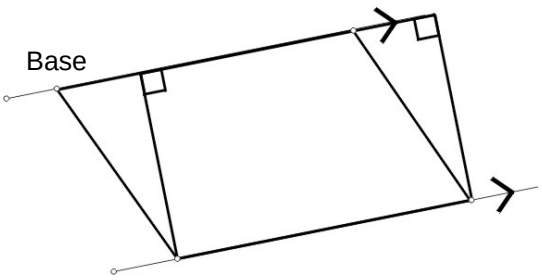


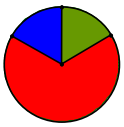
Constructing heights to different bases – example





Visual representation of the equality of areas of a parallelogram and rectangle





Description

- Develop and apply the formula for finding the area of a triangle.

Materials

- grid paper
- *The Geometer's Sketchpad*®
- BLM 14.1

Assessment Opportunities

Minds On ...

Whole Class → Connections

Students find the area of their textbook cover and estimate areas of various shapes in the classroom, e.g., desktop. Ask: How does the area of your desktop compare to the area of your textbook cover? Collectively create a chart listing several items with area of 1 m^2, 1 cm^2, … including various shapes.

Action!

Pairs → Guided Exploration

On grid paper, pairs of students draw five shapes: a square, a rectangle, a rhombus, and two different parallelograms. The partners consult and agree on the areas of all ten shapes. They cut out the shapes, and fold or cut each shape into two congruent triangles and predict the area of the triangles. Students record their observation of the relationship between the area of a triangle and a quadrilateral.

Whole Class → Connections

Discuss the students' observations and determine a general rule for the relationship, such as the triangle area is always half of the partner quadrilateral. It is important that the rule be stated in students' own words. Draw a triangle and invite volunteers to use dotted lines to complete a "partner quadrilateral," noting base and height. Encourage various solutions, and predict and calculate areas. Provide written solutions using good form for last few examples used.

Pairs → Investigation Connections

Curriculum Expectations/Performance/Rubric: Assess students' problem-solving skills and appropriate use of mathematical language either by listening to their discussion or by requiring them to submit a written summary of the work.

Provide small groups or pairs of students with BLM 14.1 on a transparency. Students add triangles of equal area and justify their additions in at least two different ways. They do this by counting squares for area, by referencing base and height, or by using partner quadrilaterals. In addition to gaining confidence in conjecturing and validating statements, students develop an understanding that two or more triangles with the same area (and the same height and base) can have different shapes. The shape is determined by size of the angles.

Alternately, provide a *Geometer's Sketchpad* ® visual demonstration of the vertex of a triangle moving along a line parallel to the base of the triangle with the measurement of area calculated for each new position.

Consolidate Debrief

Whole Class → Discussion

Students suggest applications of the area of triangle, e.g., space needed on a cookie sheet to bake a turnover.

Students work on selected questions requiring recognition of base and height and calculations of area of triangles and quadrilaterals.

Home Activity or Further Classroom Consolidation

Concept Practice

Draw shapes to illustrate situations where:

- the area of the triangle is $\frac{1}{4}$ the area of the parallelogram;
- the area of the triangle is two times the area of the parallelogram;
- the area of the triangle is equal to the area of the parallelogram.

In each case, draw a diagram of the shapes to explain the results.

Partner quadrilaterals are quadrilaterals constructed by a triangle and a transformation of the triangle (reflection, translation, rotation). Since they are comprised of two congruent triangles, the area will always be double the area of one triangle.

Triangles may have multiple partner quadrilaterals.

Some students may calculate area by counting the squares on the grid paper, while others may use known formulas, such as A = l × w.

While students may be familiar with the formula for area, the emphasis should be on inquiry and communication.

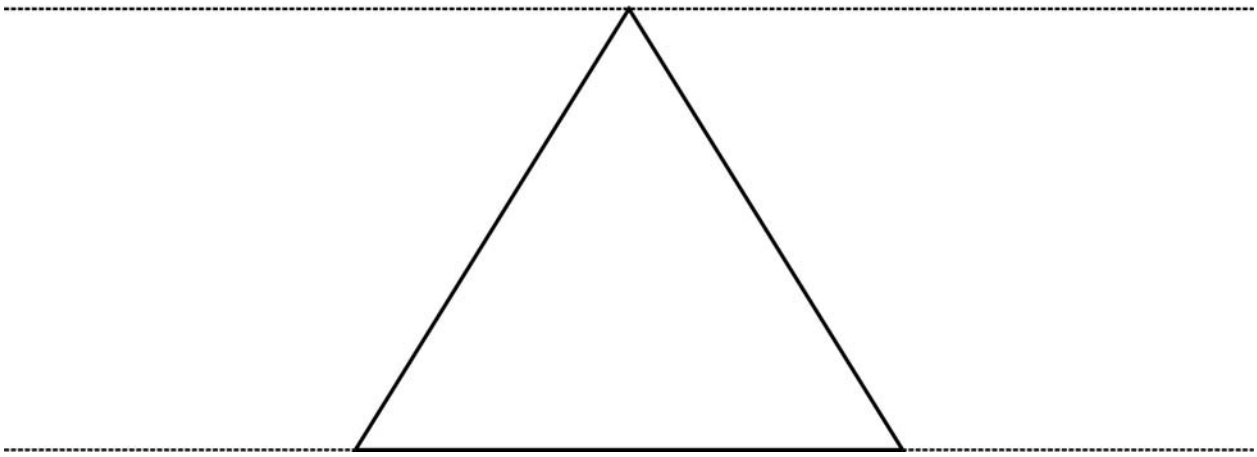
If computers are available, students can access the interactive lesson http://illuminations.nctm.org/mathlets/IGD_areas/index.html

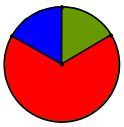
Select a student to add vocabulary to the Word Wall.

14.1: Triangles

Name:
Date:

Draw other triangles that have the same area as the one constructed below. Justify that the areas for your triangles are the same. Use at least two different methods for justification.





Description

- Use formulas and diagrams to find the area and perimeter of irregular polygons (composite figures).

Materials

- Tangram sets
- grid paper
- overhead grid
- BLM 15.1, 15.2

Assessment Opportunities

Minds On ...

Whole Class → Sharing

Selected students share their diagrams from Day 14. Reinforce that for partner shapes, the triangle's area is half the quadrilateral's area.

Pairs → Peer tutoring

Pairs of students use tangrams from Day 2 to create figures (BLM 2.7). Students predict and calculate the area of each figure in square centimetres.

Curriculum Expectations/Paper and Pencil/Marking Scheme: Use a short quiz to assess students' understanding of calculating area for various figures.

Students may suggest many processes. They will likely involve either subdividing the figure into familiar shapes or extending the figure into a quadrilateral and subtracting missing area.

Action!

Whole class → Guided Problem Solving

Curriculum Expectations/Observation/Mental Note: Ask students to suggest ways of finding the area of each shape on BLM 15.1. Observe the ways students subdivide the various shapes and present their illustrations on the board.

Guide students to see different ways to calculate areas of composite figures. Students should label their diagrams. Model the processes and form of written communication using one of the shapes on BLM 15.1. Discuss when each process may be most appropriate. Ask: Could you use symmetry to find the area of any of the shapes?

Subsequent steps will include:

- taking needed measurements;
- representing symbolically, substituting into formulas, then computing;
- noting appropriate units.

Think/Pair/Share → Practice

Using grid paper, partners create a composite figure together; subdivide and find areas individually and compare results. Encourage partners to find different ways.

Individual → Differentiated Practice

Assign selected questions from BLM 15.1 to students who have grasped the concept. Provide students who require more direction with BLM 15.2.

Consolidate Debrief

Individual → Response Journal

Students make entries in their math journals based on prompts such as:

- I can tell area and perimeter measurements apart by…
- Areas of triangles and quadrilaterals are related by…
- The area of composite figures can be calculated…
- When finding area of shapes without right angles…

Pose the question: Does it make sense to add the perimeters of the parts of a composite figure together to find the total perimeter? Write an explanation to communicate your thinking. Students share responses.

Once most students in the class appear ready to demonstrate proficiency at finding perimeter and area of composite figures, assess using a version of Developing Proficiency found on page 9 of the *Developing Perimeter and Area Formulas* package.

Select a student to add vocabulary to the Word Wall.

Home Activity or Further Classroom Consolidation

Locate some composite figures for which you could find the area and perimeter, e.g., lawn, carpet in a non-rectangular room.

Decide how to explain the difference between area and perimeter to someone at home and ask them to report on the clarity of your explanation.

Complete the Extend Your Thinking based on the curriculum expectation that you estimate and calculate the perimeter and area of an irregular two-dimensional shape.

See *Developing Perimeter and Area Formulas* – Section 2 p. 12.

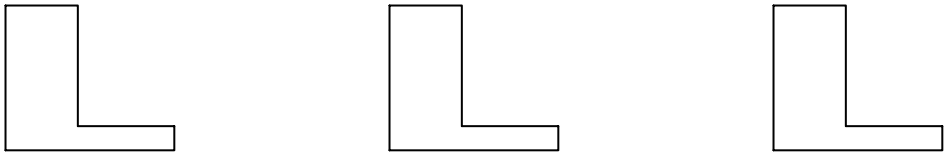
Concept Practice

15.1: Finding the Area of Composite Figures

Name:
Date:

Subdivide each shape into shapes for which you know an area formula. Do this in more than one way.

1.



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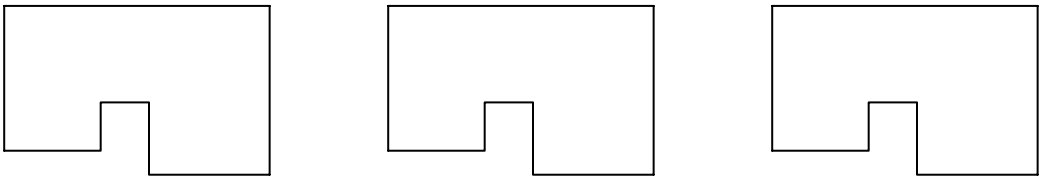

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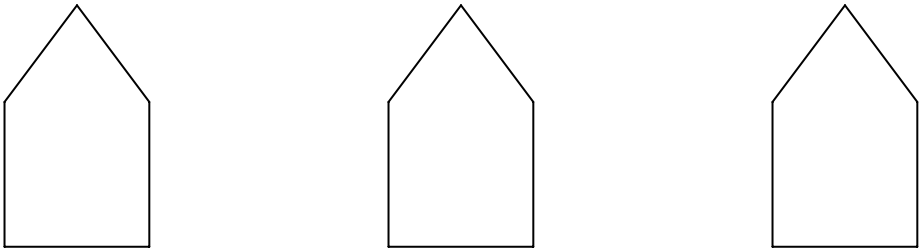

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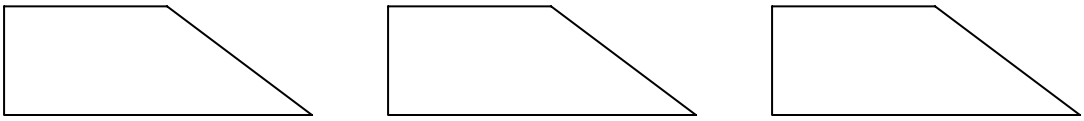

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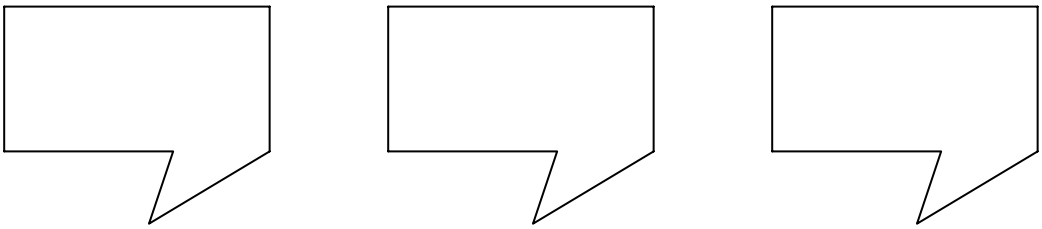

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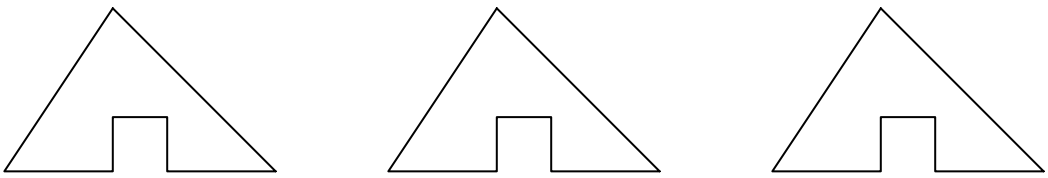
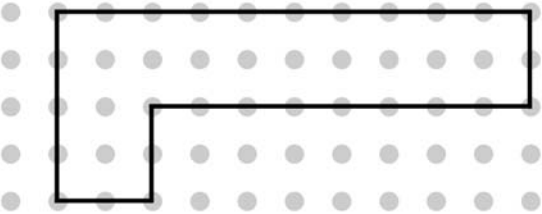


15.2: Finding Area of Composite Figures

Name:
Date:

1



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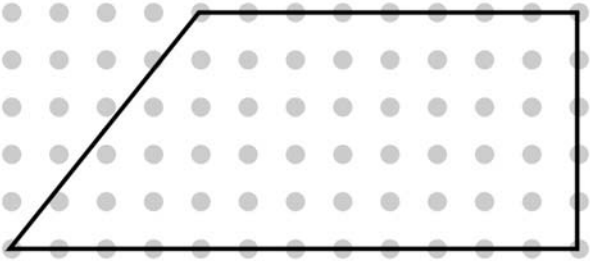

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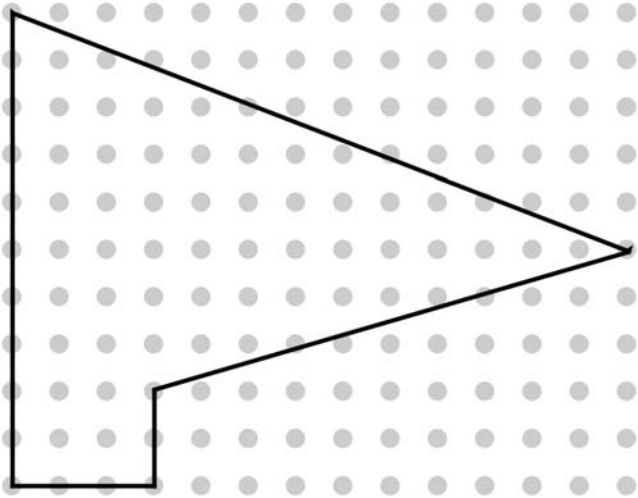

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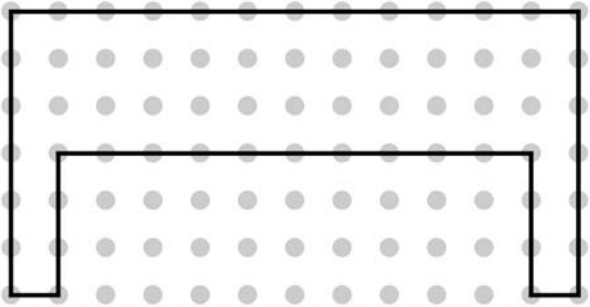
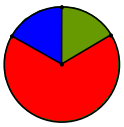


Find the area of this shape by:

a) visualizing the addition of subdivisions
b) visualizing the subtraction of areas
c) using symmetry



Description
- Develop a formula for finding the area of a trapezoid.

Materials
- BLM 16.1

Assessment Opportunities

Minds On ...

Whole Class → Sharing
Discuss and clarify the task on BLM 16.1.

Area of a trapezoid is in Term 3 cluster of expectations.

Pairs → Think/Pair/Share
Give students two minutes to think about and to record independently the process that they will use to find the rule for the area of a trapezoid. Students share strategies with a partner. Using a different colour, students record any changes they wish to make in their process.

Students should keep their work for Term 3.

Generate a list of trapezoids that students discovered for reference in Term 3.

Action!

Individual → Assessment Activity
Curriculum Expectations/Investigation/Checkbric: Focus this assessment on communication and inquiry, as well as patterning skills (BLM 16.2).
Students work through BLM 16.1, using manipulatives and materials, as appropriate. They state a rule and provide justification for their conjecture. It is more important that students use the inquiry process established in the previous lessons than that they generate the usual form for the rule or formula.

When returning graded work to students, consider photocopying samples of Level 3 and 4 responses with student names removed. Select and discuss with the class, samples that show a variety of strategies.

Consolidate Debrief

Whole Class→ Sharing
Collect student work. Students discuss the processes they used and rules that they discovered.

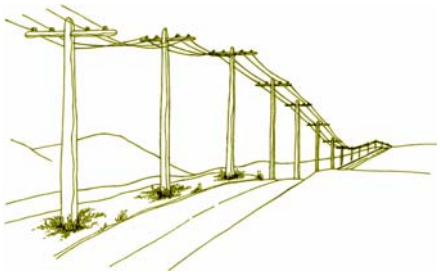
Home Activity or Further Classroom Consolidation
Identify as many trapezoids as possible in your home, school, and community. Answer "Is This Always True?"

Application

See page 18, *Developing Perimeter and Area Formulas* content-based package.

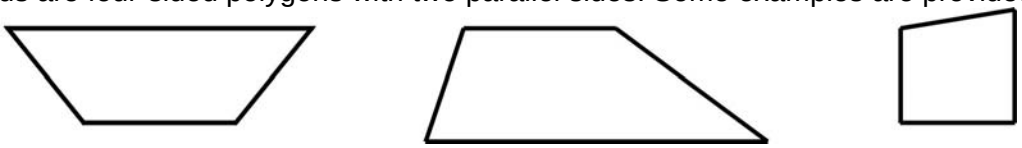
16.1: Area of a Trapezoid

Name:
Date:



Your company has been hired to seal paved highways. Sealant is applied in trapezoidal sections to ensure bonding. As there are curves and intersections, the trapezoids change size and shape for each area. Engineers need to determine the amount of sealant required to cover any trapezoidal area.

Trapezoids are four-sided polygons with two parallel sides. Some examples are provided:



Task

Determine a rule the engineers could use to calculate the area of any trapezoid.

Suggested methods include:

- Use pattern blocks to construct various trapezoids that are then sketched on dot paper.
- Draw several trapezoids on dot paper and determine their areas.
- Construct a variety of trapezoids and take useful measurements for calculating the area.
- Cut out the trapezoids and cut them further into basic shapes, like squares, rectangles, and triangles.

Record any numerical data that may help you identify patterns in an organized fashion.

Describe how to find the area for any trapezoid. Express your rule as clearly as possible, using words, pictures, and symbols.

Your process and communication are important for assessment purposes.

16.2 Assessment Tool: Area of a Trapezoid

Note: Since this performance task is to be assessed for inquiry, it is important that students not have been introduced to finding the area of the trapezoid previously. Students should not be expected to develop the standard form for the formula for the required area. Rather, any form, e.g., verbal, symbolic, or diagrammatic, for the area should be given full credit.

Mathematical Process (Category)	Criteria	Below Level 1	Level 1	Level 2	Level 3	Level 4
Knowing Facts and Procedures (Application)	Correctness	Use a marking scheme.				
Reasoning and Proving (Problem Solving)	Gathering appropriate measurement data connected to area of trapezoids					
	Connection between the pattern identified in areas of trapezoids and the areas of particular trapezoids					
Demonstrating Understanding (Understanding of concepts)	Appropriateness of the strategy chosen for calculating areas of trapezoids					
	Completeness of the strategy chosen					
Communicating (Communication)	Use of conventions accurately, effectively, and fluently					
	Clarity of rule					

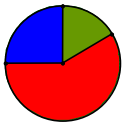
Lesson Outline: Days 17 - 22 Grade 7

BIG PICTURE

Students will:

- connect the relationship between 3-D figures and their 2-D shapes;
- connect 3-D figures to their various nets and views;
- design and build a structure using polyhedrons;
- develop appropriate inquiry skills.

Day	Lesson Title	Description	Expectations
17	Review of Polyhedrons	• Review understanding of polyhedrons and their nets. • Construct Platonic solids.	7m48 CGE 3c
18	3-D Figures and Number Cubes	• Draw nets of cubes. • Classify and draw front, side, and top views of familiar 3-D figures.	7m52, 7m54 CGE 3c, 5a
19	Building a Collection of 3-D Figures	• Construct different prisms and pyramids.	7m56 CGE 4c
20 21	Geo Visions Design and Construction Project	• Design and build scale models of geometric structures as a project. • Use nets to construct 3-D structures. • Present the models and design analysis to the class.	7m47, 7m48, 7m49, 7m52, 7m53, 7m54, 7m55, 7m56 CGE 2c, 4c
22	Performance Task	• Sketch views of polyhedrons. • Investigate the relationship that exists between the number of faces, edges, and vertices for polyhedrons and Platonic solids.	7m74, 7m80 CGE 3c, 5g



Description

- Review understanding of polyhedrons and their nets.
- Construct Platonic solids.

Materials

- BLM 17.1, 17.2
- plain, isometric, or orthographic paper
- polyhedron manipulatives

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Curriculum Expectations/Question & Answer/Mental Note: Students identify which one of two model-making activities is most appropriate for them based on their prior knowledge of nets.

Using a collection of polyhedrons and nets, show students a net and ask what the corresponding 3-D figure would be. Show a polyhedron and ask what shapes would make up its net. Students describe the faces of a rectangular prism.

Explain that some students will review and revisit construction of polyhedra and nets. Other students will extend prior knowledge through an investigation of the traits of Platonic solids (BLM 17.2).

Action!

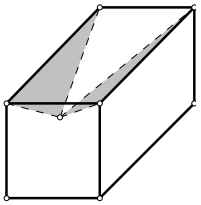
Small Groups → Differentiated Instruction

Students needing review: Using the shapes on BLM 17.1 and 17.2 as patterns, make available multiple copies of each shape or reproduce the shapes on tagboard for students to use as patterns to cut out the required number of shapes for constructing the polyhedron. Depending on the manual dexterity of students, tabs on the sides of the shapes may make construction easier. Students tape the edges of the shapes together to construct as many different polyhedrons as possible (concave as well as convex). They can combine different polyhedrons to make irregular and unusual polyhedrons and name and describe the solid they constructed.

Circulate among the groups, discussing the process with them, listening for proper terminology and evidence of understanding. Ask higher-order questions such as: Which polyhedron has the smallest surface area? the greatest? How could your collection of polyhedrons be grouped or ordered? Is it possible to build polyhedrons with 4, 5, 6…12 faces? Name the solids you are able to construct.

Students ready to extend their prior knowledge: Students build the Platonic solids and investigate why there are only five possible solids that satisfy the definition of a Platonic solid. (BLM 17.2)

An example of a convex polyhedron



If polydron manipulatives are available, they could be used instead (BLM 17.2), especially for students who lack manual dexterity.

Consolidate Debrief

Whole Class → Group Presentations

Students display and describe orally their polyhedron constructions or the results of their investigation of Platonic solids.

Select a student to add vocabulary to the Word Wall.

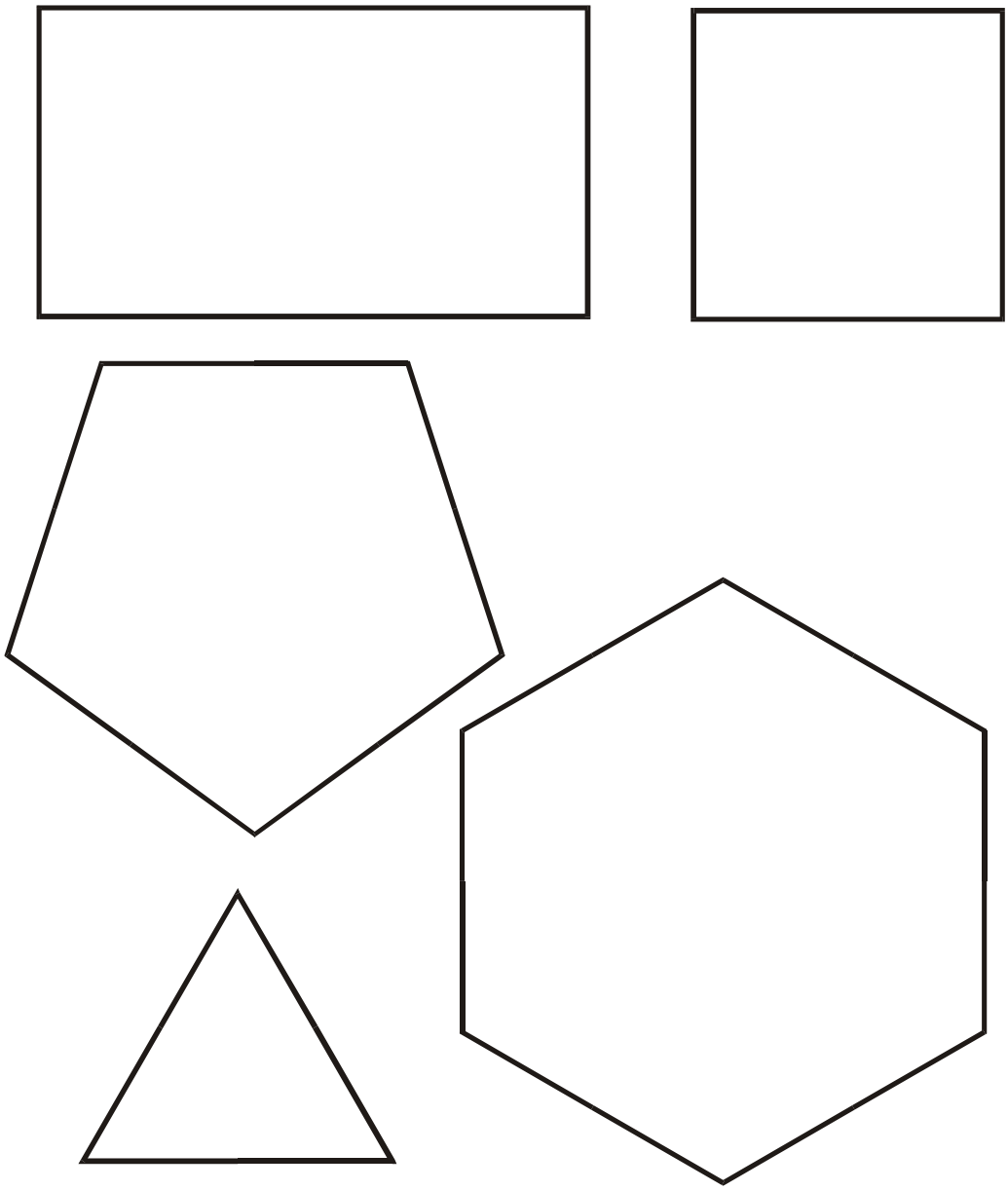
Home Activity or Further Classroom Consolidation

- Decorate at least one polyhedron as a possible 'package' for a product, e.g., a fancy box for a perfume bottle or chocolate, a holiday ornament, etc. Write a detailed description of how to construct it, and an advertisement for the product.
- Look for an 'unusual' polyhedron at home. Write a poem, song, or rap about its structure.
- Collect pictures of various polyhedrons to put into a collage. Write a brief explanation of the characteristics of several of the polyhedrons you selected.

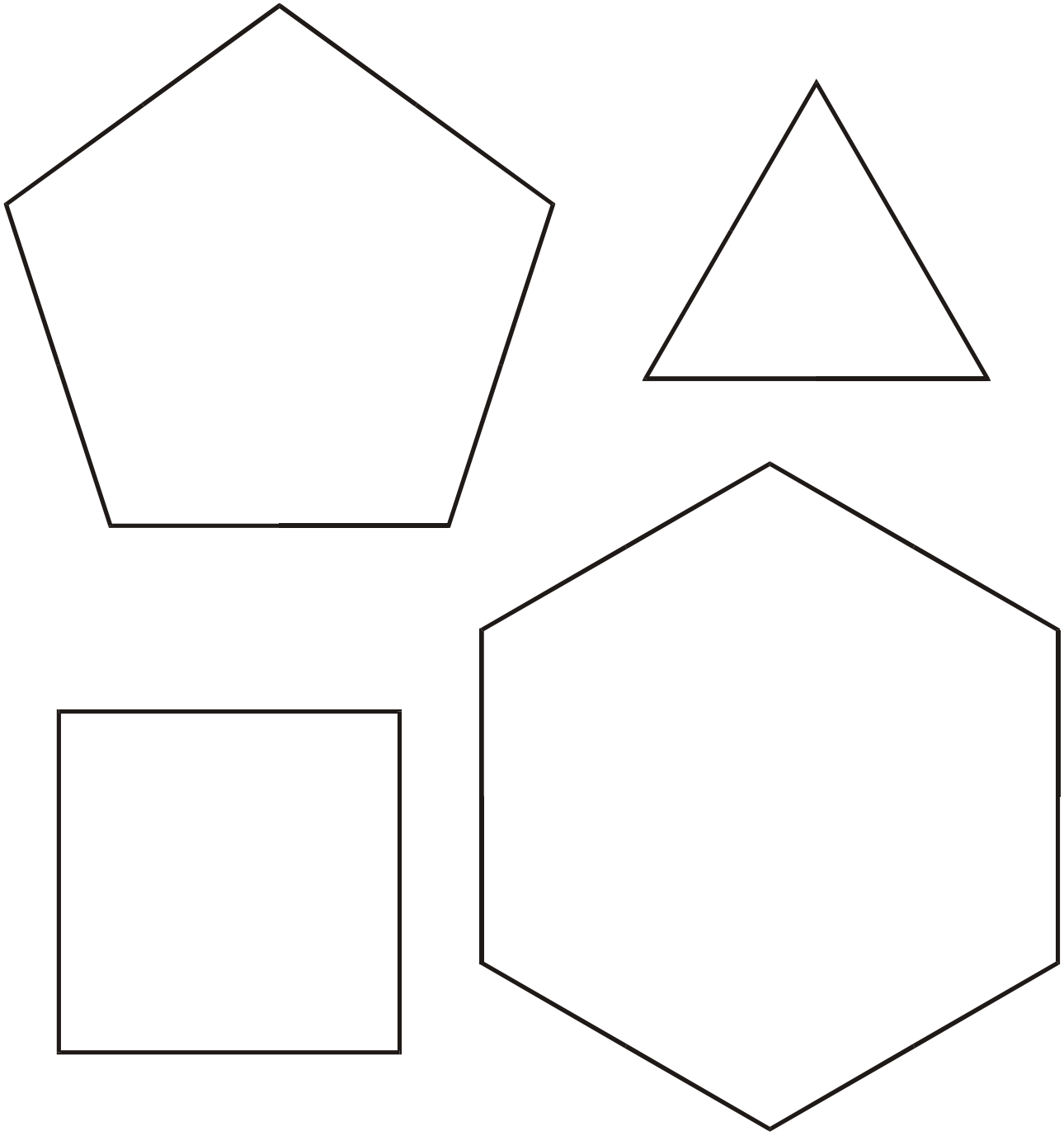
The tasks provide cross-curricular linkages. It may be appropriate to offer students a choice as to which task they do.

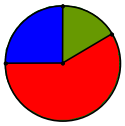
Differentiated
Application
Concept Practice

17.1: Building Solids – Teacher



17.2: Building the Platonic Solids – Teacher





Description

- Draw nets of cubes.
- Classify and draw front, side, and top views of familiar 3-D figures.

Materials

- polyhedrons
- cube-shaped box
- graph or dot paper
- BLM 18.1
- Platonic numeric solids

Assessment Opportunities

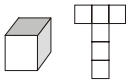
Minds On ...

Whole Class → Guided

Curriculum Expectations/Question & Answer/Mental Note: Assess students' use and understanding of mathematical language.

Show a variety of small polyhedrons. Ask:

- What is the name of this 3-D figure?
- What do we mean by congruent?
- How many congruent faces does this (polyhedron) have?
- How many edges does it have? … vertices?
- What 2-D shape would you sketch as the front view of this 3-D figure? … the top view?



Use a cube-shaped box that has been cut along its edges to create a net. Students compare the number of congruent squares on the net to the number of congruent square faces on the 3-D figure.

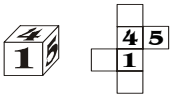
Translated and rotated nets are considered to be the same net.

Action!

Pairs → Investigation

Students estimate the number of nets for a cube and then find all the possible different nets for a cube. Students sketch each different net on dot or graph paper. Ask: How can we confirm or refute our estimate?

Invite students who complete the task to sketch one of the 11 nets on the board or on a transparency. Students check their solutions against the board sketches.



Challenge students to arrange the numbers 1 through 6 on the faces of the net so that opposite faces have a sum of 7 (BLM 18.1).

Students cut out one of their nets from large-squared graph chart paper and form it into a cube. They write the numbers on the sides of the cube before re-opening the net.

Consolidate Debrief

Whole Class → Student Presentation

Some students explain their organized methods of determining all the possible arrangements for the numbers. How can we be sure that we have counted all of the possible arrangements?

Select a student to add vocabulary to the Word Wall.

Individual → Independent

Demonstrate the task on BLM 18.1, using the triangular prism packaging for some chocolate bars and talk about how it would look from different perspectives.

Home Activity or Further Classroom Consolidation

Complete worksheet 18.1.

Exploration
Skill Drill

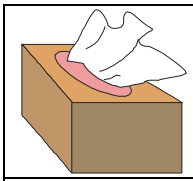
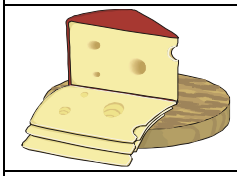
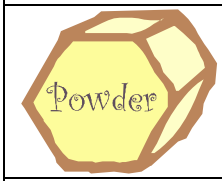
Challenge: Orient the numerals on the nets so that they fold up as on a regular number cube.

Determine the sum of the numbers on opposite faces of each of the Platonic solids. Provide some reasoning for your findings.

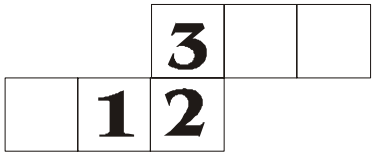
18.1: Sketching 3-D Figures

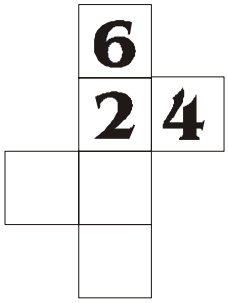
Name:
Date:

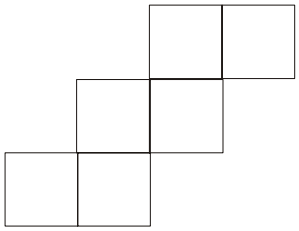
Name each 3-dimensional figure illustrated below. Sketch its front, side and top views.

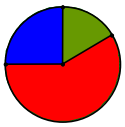
	3-D sketch	Front View	Side View	Top View
				
				
				
				
				
				

On the net of each cube, place the numbers 1, 2, 3, 4, 5, and 6 so that opposite faces have a sum of 7.









Description

- Construct different prisms and pyramids.

Materials:

- BLM18.1
- Platonic numeric solids

Assessment Opportunities

Minds On ...

Whole Class → Guided

Use large cut-outs of the 3 nets for a cube on BLM 18.1 to discuss the students' responses and to review vocabulary.

Curriculum Expectations/Question & Answer/Mental Note: Show a variety of familiar polyhedrons one at a time, and have students predict what the net of each figure would look like. Encourage students to use appropriate mathematical terminology. For example, ask, What would the net of this chocolate bar package look like? Open the chocolate bar package into its net to illustrate the 3 congruent rectangles and 2 congruent triangles.

Demonstrate the nets for a prism and a pyramid pair, e.g., a cube and a square-based pyramid. Ask, What are the similarities and differences between prisms and pyramids? Compare the number of congruent shapes on the net to the number of congruent faces on the 3-D figure.

Product packaging: A chocolate bar package having three congruent rectangles and two congruent triangles and an octagonal prism cleaning cloth box.

Some nets are in Impact Math: *Geometry & Spatial Sense* pages 15-20.

Action!

Pairs → Tactile Activity

Each pair constructs at least one of the 16 polyhedrons - triangular, square, rectangular, pentagonal, hexagonal, and octagonal prisms; triangular, square, rectangular, pentagonal, hexagonal and octagonal pyramids; tetrahedron; octahedron; icosahedron; and dodecahedron, from nets using polyhedrons, paper/cardboard, TABS+ software.

Ask:

- How could you use ratios (or similar triangles) to make the net of a polyhedron larger or smaller?
- How many faces of a polyhedron share the same edge?
- Is it possible to create a polyhedron with two different sets of congruent faces?

Circulate among the groups, discussing the process and, listening for proper terminology and evidence of understanding. Assist students who have difficulty manipulating and gluing the nets.

TABS+ software [TABS+ is a Ministry licensed program for designing nets. It is user-friendly and allows students to experiment with simple and complex 3-D figures in a short period of time. Instructions for using the software are included in this resource, in the section containing manuals.]

Note: A complete class set of these 16 polyhedrons is needed for the summative assessment task.

Consolidate Debrief

Whole Class → Student Presentation

Select students to present their polyhedrons based on variety and quality of figures.

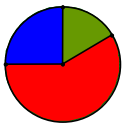
Curriculum Expectations/Quiz/Marking Scheme: Students demonstrate appropriate use of language and identification of correspondence between nets and 3-D figures.

Select a student to add vocabulary to the Word Wall.

Application

Home Activity or Further Classroom Consolidation

Conduct a 3-D figure scavenger hunt searching for figures to add to the classroom collection. (You may find regular or irregular figures, but they should have flat, not curved, faces.)



Description

- Design and build scale models of geometric structures as a project.
- Use nets to construct 3-D structures.
- Present the models and design analysis to the class.

Materials

- Impact Math: *Geometry and Spatial Sense*
- Geo Visions – Design and Construct
- BLM 20.1

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Discuss some famous sites known for their geometric design, e.g., the pyramids in Egypt, the Eiffel Tower, Geodesic Dome. Which 3-D figures are most often used in construction?

Why are triangles used so frequently in construction?

Read the task on BLM 20.1 with students. Review the rubric that will be used.

Geo Visions is a unit written for Grade 6/7 by York Region Teachers and available on the Ontario Curriculum Unit Planner.

Action!

Individual → Model Making

Curriculum Expectations/Performance/Rubric: Assess students on their design and response to questions 1-4 on BLM 20.1.

Students design and build scale models of castles, cars, people, animals, abstract geometrical structures, etc. using a variety of three-dimensional figures that they construct from nets. Nets can be made using any of the methods from Day 19.

Scaffold: Students can manipulate different combinations of the polyhedrons created in Day 19 to help them visualize the type of structure they might choose to design.

Consider integrating expectations from the Grade 7 Science strand Structures and Mechanisms with this activity.

If questions 1-4 are being used as summative assessment data, the questions must be completed under the teacher's supervision so that the level of independence demonstrated by the student can be assessed.

Consolidate Debrief

Individual → Reflection

Learning Skills/Self Assessment/Checklist: Students complete a self-assessment of their work using the checklist on BLM 20.1.

- How much of the required work have I completed?
- How much must I finish to be ready for a presentation?
- What clarifying questions must I ask to be able to complete the work?

Application

Home Activity or Further Classroom Consolidation

Complete the building of your structure.

20.1: Design/Construction Project

Task

You are a designer with the Geo Visions Company. Geo Visions specializes in creating and constructing geometric structures, e.g., office buildings, art galleries, castles, houses, bridges, etc. They also design geometric vehicles, e.g., cars, trucks, boats, or geometric sculptures, e.g., people, animals, abstract sculptures.

You have been hired by the city to design and construct _____
(Select a type of structure, vehicle, or sculpture.)

You will need to submit a model and an analysis of your design to the city planners during the week of _____ (Write in the due dates.).

Details of the Design Project

Your model must:

- be constructed with 3-D geometric figures (each figure constructed from a net)
- fit on a base 30 cm by 30 cm
- include at least 2 different types of prisms
- include at least 2 different types of pyramids
- include at least 2 other different types of 3-D figures.

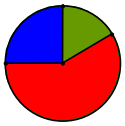
Your design analysis must include:

- nets of the 2 different types of prisms (label dimensions of each face)
- net of the 2 different types of pyramids (label angle measure of each face; label dimensions of each face)
- sketch of front, back, and side views of your structure or sculpture
- a description for each of the prisms and pyramids mentioned above (number of faces, edges, vertices; classification of faces, e.g., rectangle, square, acute isosceles triangle, etc.; tell which faces are congruent.).

To show the city planners that you have a good understanding of 2-D and 3-D geometry, include answers for the following.

1. If Cube A has dimensions 2 cm by 2 cm by 2 cm and Cube B has dimensions 4 cm by 4 cm by 4 cm, which faces are congruent and which faces are similar? Explain. Include a diagram with your explanation.
2. Draw the net for a rectangular prism that has dimensions of 6 cm by 4 cm by 2 cm. Include the dimensions of each face on your net.
3. Can a quadrilateral contain 3 obtuse angles? Explain.
4. Is this statement always true, sometimes true, or never true? Explain.
   “A right-angled triangle is isosceles.”

Hand in your model, your design analysis, and answers to the questions. Use the check boxes on this page to make sure you have completed everything.



Description

- Design and build scale models of geometric structures as a project.
- Use nets to construct 3-D structures.
- Present the models and design analysis to the class.

Materials
see Day 20

Assessment Opportunities

Minds On ...

Whole Class → Sharing
Discuss and share students constructions started on Day 20:
- How did you plan your design before you began cutting out nets?
- Did you model your design on any known architectural design?
- What problems did you encounter and how did you resolve them?

Action!

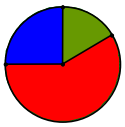
Individual → Construction of Concrete Models
Continue and complete the design project.

Consolidate Debrief

Whole Class → Presentation
Curriculum Expectations/Observation/Checklist
Students describe how their design meets all the requirements of the project. A display could be set up in the school for all students/parents to examine.

Concept Practice

Home Activity or Further Classroom Consolidation
Review your work from this unit and prepare for the Performance Task.



Description

- Sketch views of polyhedrons.
- Investigate the relationship that exists between the number of faces, edges, and vertices for polyhedrons and Platonic solids.

Materials

- BLM 22.1
- 22.2 Assessment Tool

Assessment Opportunities

Minds On ...

Whole Class → Guided

Show one of the polyhedrons. Ask students to identify the figure. Together, count the number of edges, faces, and vertices on the figure.
Pose this question: How do you think the number of edges, faces, and vertices of a polyhedron are related?
Lead a brief class discussion to ensure understanding.
This assessment activity requires that students have several different polyhedrons available to use as models as they complete the investigation.

The relationship is F + V = E + 2, called Euler's Theorem, although students need not present it in this form and are not responsible for knowing the name of the formula.

Action!

Individual → Investigation

Curriculum Expectations/Test/Rubric: Discuss the Assessment Rubric with students before they begin. (22.2 Assessment Tool)

The focus for this assessment is on the students' ability to complete an investigation and to form appropriate conjectures based on organized and thoughtful data collection.

The investigation leads students to formulate an hypothesis about the relationship among the number of edges, faces, and vertices found in prisms and pyramids. Students then test their hypothesis on the Platonic solids. (BLM 22.1)

Students use the concrete models to sketch front, side, and top views of the polyhedrons, and then count the number of faces, edges, and vertices.

Ensure appropriate sharing of the models and independent investigation of the relationship. Circulate around the classroom and observe students' work.

Should you take a particular polyhedron model to a particular student?
Do certain students need prompts to speed up their data collection?
How will you keep a record of the supports you provide to individual students?

Check that students have accurately completed question 1b) on the BLM 22.1 before completing the remaining questions.

Students who complete the activity early can test their hypothesis on some of the concave and convex polyhedrons created during Day 17.

Reference: Assessment. Accommodations – Section 2 p. 24

Classroom collection of polyhedrons built during the unit should be displayed and readily available around the classroom

Consolidate Debrief

Whole Class → Student Discussion of findings

Students discuss their findings or conduct a debriefing with them when their assessments are returned.

Home Activity or Further Classroom Consolidation

Reflection

In your math journal, reflect on the task. Identify your personal strengths, areas of weakness, and some next steps for your mathematics studies. Your comments should focus on your understanding and use of geometric language and the inquiry process.

22.1: Investigating the Properties of Polyhedrons

Name:
Date:

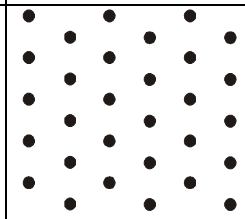
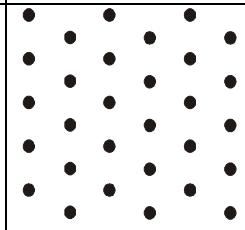
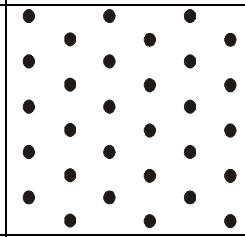
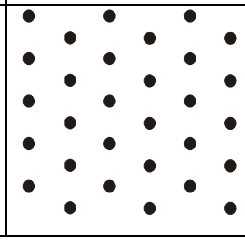
1. A polyhedron is a 3-dimensional figure. Examine models of these polyhedrons:

rectangular prism **square-based pyramid**

triangular prism **pentagonal pyramid**

hexagonal prism

a) Draw the 3-dimensional, front, side, and top views of each polyhedron.

	3-D sketch	Front View	Side View	Top View
rectangular prism				
square-based pyramid				
triangular prism				
pentagonal pyramid				

22.1: Investigating the Properties of Polyhedrons (continued)

b) For each of the five polyhedrons you examined, determine the number of faces (F), the number of vertices (V) and the number of edges (E). Do not fill in the conjecture column yet.

	F (Number of Faces)	**V** (Number of Vertices)	**E** (Number of Edges)	**Conjecture**
Rectangular Prism				
Triangular Prism				
Hexagonal Prism				
Square Pyramid				
Pentagonal Pyramid				

c) Examine the values you have recorded for F, V and E. Identify patterns that you see within each column, e.g., How does F change as the shape changes?

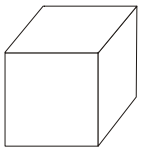
d) Make a conjecture about how F, V and E are related to each other. Look for patterns across the table, within each row.

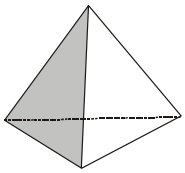
e) Give this conjecture a name: (*e.g., John's Theory, Moira's Hypothesis, O' Reilly's Idea*). You investigate the accuracy of your conjecture in question 2b.

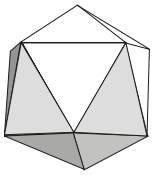
22.1: Investigating the Properties of Polyhedrons (continued)

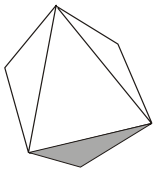
2. A Platonic solid is a regular polyhedron that has all faces congruent and each face is a regular polygon. (The Platonic solids are named after Plato, a famous mathematician.)

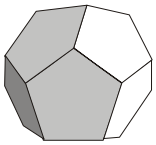
There are 5 Platonic solids:

cube 

tetrahedron 

icosahedron 

octahedron 

dodecahedron 

a) What regular polygons form the faces of each of the Platonic solids?

b) Examine the number of faces, vertices and edges of the Platonic solids.
Is your theory about the relationship of F, V and E true for these Platonic solids?
Justify your answer.

c) What conclusions can you make about the accuracy of your theory? Justify your conclusion.

22.1 Assessment Tool: Investigating the Properties of Polyhedrons

Mathematical Process (Category)	Criteria	Below Level 1	Level 1	Level 2	Level 3	Level 4
Making Connections (Understanding of Concepts)	Quality of fit of relationship among F, V, and E	- little or no evidence fit	- narrow fit	- moderate fit	- broad fit	- very extensive fit
Reasoning and Proving (Problem Solving)	Makes convincing arguments as to why an octahedron is a Platonic solid	- no argument present	- major omission in the argument	- omission in the argument	- thorough argument	- complete and extended argument
	Explores whether or not Platonic solids satisfy the F/V/E relationship	- does not gather data appropriate to the question	- gathers little of the data appropriate to the question	- gathers some of the data appropriate to the question	- gathers the data appropriate to the question	- gathers a wide variety of data appropriate to the question
	Makes conclusions about whether Platonic solids satisfy the F/V/E relationship	- conclusion missing or not connected to the data gathered and the question posed	- conclusion weakly connected to the data gathered and the question posed	- conclusion moderately connected to the data gathered and the question posed	- conclusion well connected to the data gathered and the question posed	- conclusion insightfully connected to the data gathered and the question posed
Communicating (Communication)	Clarity of Explanation	- explanations are unclear or confusing	- have limited clarity	- have some clarity	- explanations are clear	- explanations are precise
	Use of Mathematical Conventions	- demonstrates an undeveloped use of conventions	- demonstrates minimal skills in use of conventions	- demonstrates moderate skills in use of conventions	- demonstrates considerable skills in use of conventions	- demonstrates a high degree of skill in use of conventions
Knowing Facts and Procedures (Application)	Completes the chart for F, V, and E correctly Identifies the faces of the Platonic solids correctly Represents the polyhedrons in all of: 3-D, front, side, and top views accurately	Use a marking scheme.				

Lesson Outline: Days 23 - 28 Grade 7

BIG PICTURE

Students will:

- connect 3-D figures to their various nets and views;
- develop the formulas for finding the surface area of a rectangular prism using nets;
- investigate the relationship between the dimensions and the volume of a rectangular prism;
- develop the formula for finding the volume of a rectangular prism using concrete materials;
- apply surface area and volume formulas to problem-solving situations involving rectangular prisms.

Day	Lesson Title	Description	Expectations
23	Surface Area of Cubes	• Draw the 3-D view of a cube and an appropriate net. • Develop and apply the formula for finding the surface area of a cube.	7m36, 7m55, 7m73, 7m76, 7m77 CGE 2a, 3c
24	Surface Area of Rectangular Prisms	• Develop and apply the formula for finding the surface area of a rectangular prism.	7m36, 7m42, 7m55, 7m73, 7m76, 7m77 CGE 2a, 4b
25 26	Volume of Rectangular Prisms	• Build rectangular prisms using manipulatives. • Develop and apply the formula for finding the volume of a rectangular prism.	7m43, 7m44, 7m45, 7m55, 7m76, 7m77 CGE 2c, 3c, 5a
27 28	Cereal Box Challenge	• Design a cereal box that meets certain specifications (a fixed volume). • Apply volume and surface area formulas for rectangular prisms. • Present designs to the class.	7m31, 7m36, 7m44, 7m45, 7m46, 7m54, 7m76, 7m77 CGE 2c, 2d, 5g

Description

- Draw the 3-D view of a cube and an appropriate net.
- Develop and apply the formula for finding the surface area of a cube.

Materials:
- isometric dot paper
- orthographic dot paper

Assessment Opportunities

Minds On ...

Whole Class → Guided Exploration

To introduce surface area show a net of a cube drawn on chart paper. Students identify and explain the connection between the area of the net and the surface area of the cube.

Students discuss with a partner what they think "surface area" means and describe how it is different from area. As a class discuss various definitions and record them on a transparency or the board. Discuss when it would be useful to determine the surface area of a rectangular prism.

Give students a few minutes to individually determine a method for finding the surface area of a cube, then facilitate sharing of various methods. Discuss similarities between the methods highlighted.

Include boxes other than rectangular prisms, such as triangular-based prism chocolate bar box, octagonal cleaning cloth box, and cylindrical salt or oatmeal boxes.

Individual → Investigation

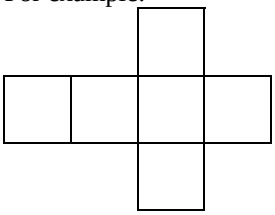
Pose the questions: What is the area of each face, if the width, length, and height are 10 cm? How could you calculate the total surface area of the cube?

Curriculum Expectations/Exhibition/Rubric: Observe students as they work through the process of answering the questions. Assess their inquiry skills (see rubric for Day 27). **Note:** This same process can be observed over the next three classes allowing the teacher to focus on one third of the class each day.

Encourage multiple approaches for finding total surface area [including area of sides + top + base]

Whole Class → Sharing

Discuss the solutions of various students. Students identify which units are used and how to properly include that information,
For example:



Students represent the relationship in a variety of ways (words, variables and numbers). All forms are equally acceptable.

Area of One Face, $A = l \times w$
Total Surface Area = $A \times 6$

Ask students to bring an empty cereal box to class for Day 27.

Select a student to add vocabulary to the Word Wall: *orthographic, isometric, net, justify*

Action!

Pairs → Visual Activity

Students use a rectangular prism (not a cube) from the class collection, measure its sides and use dot paper to draw a net. They calculate the surface area of the box. Encourage students to use a variety of nets and methods.

When returning graded work to students, consider photocopying samples of Level 3 and 4 responses with student names removed. Select and discuss, with the class, samples that show a variety of strategies.

Consolidate Debrief

Whole Class → Reflection

Students share their explanations of how they found the surface areas.

Home Activity or Further Classroom Consolidation

Concept Practice

Write a description that you could use to find the surface area of any rectangular prism. In your math journal explain your approach to finding the surface area.

Description

- Develop and apply the formula for finding the surface area of a rectangular prism.

Materials

- boxes and nets (Day 23)
- isometric dot paper
- orthographic dot paper
- *The Geometer's Sketchpad*®

Assessment Opportunities

Minds On ...

Whole Class → Student Presentation

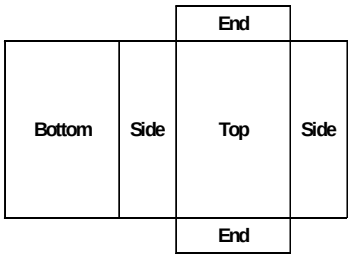
Selected students read their math journal entry explaining their approach to calculating the surface area of their box. Ask: Are there other ways to express a valid process for calculating total surface area?

Encourage students to use descriptive formulas until they are ready for symbolism.

Action!

Small Groups → Conferencing

Students use their solutions for calculating surface area to develop an algebraic formula for the surface area of a rectangular prism. They record the formula on chart paper.



Area = $l \times w$

Surface Area =
top + bottom + 2 sides + 2 ends
(descriptive formula)

Students should use appropriate mathematical notation when presenting their solution. Strive for an all-inclusive formula as in the one shown.

If computers are available, students use *The Geometer's Sketchpad*® to sketch a prism, predict the surface area (SA), use the "measure" tool to verify, and explore a variety of alternate prisms.

Curriculum Expectations/Observation/Rubric: Observe students as they work through the process of answering the questions. Assess their inquiry skills (see 27.2 Assessment Tool, Day 27).

Some students may need to scaffold their solutions, e.g.,
SA of top and bottom = 2 (b x h)
SA of two ends = 2 (b x h)
SA of two sides = 2 (b x h)
Total SA of rectangular prism = __ + __ + __
= ____ units^2

Consolidate Debrief

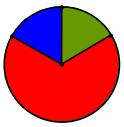
Whole Class → Student Presentation

Students present their formulas. To assist students as they move towards symbolic representation, discuss how the various representations convey the same information or result in the same answer. Highlight advantages to symbolic representation.

Home Activity or Further Classroom Consolidation

Skill Drill

Complete the following questions for additional practice: (The teacher inserts text references.)



Description

- Build rectangular prisms using manipulatives.
- Develop and apply the formula for finding the volume of a rectangular prism.

Materials

- linking cubes
- isometric dot paper
- orthographic dot paper
- BLM 25.1

Assessment Opportunities

Minds On ...

Whole Class → Guided

Show students a cube. If the length of one side is 1 unit, what is the surface area of one face? (1 unit^2) What is the volume? (1 unit^3.)

Distribute BLM 25.1 and invite students to ask clarifying questions.

Link to lessons on patterning.

Action!

Pairs → Investigation

Have the students construct the first two cube objects and complete the first two rows for each chart on BLM 25.1. Ensure that all students understand how to complete the chart correctly before they continue. Students construct each of the cube objects before filling the values in on the chart. They think about what is changing each time a row is added.

Curriculum Expectations/Observation/Checklist: Provide some support for students who are having difficulty generating rules depending on the number of the term. Refer back to the patterning activities completed previously in the term. Students must be allowed to develop their own rules based on the patterns they observe.

Identify any students who are experiencing difficulty in developing a patterning rule. Students who were successful at developing patterning rules on Days 5-9 may need to be reminded to apply that knowledge. Other students may need further instruction.

Students gain better insight into the construction of a general rule or equation by actually constructing the objects.

Consolidate Debrief

Whole Class → Student Presentation

Summarize the results of the investigation on a class chart.

Reflection

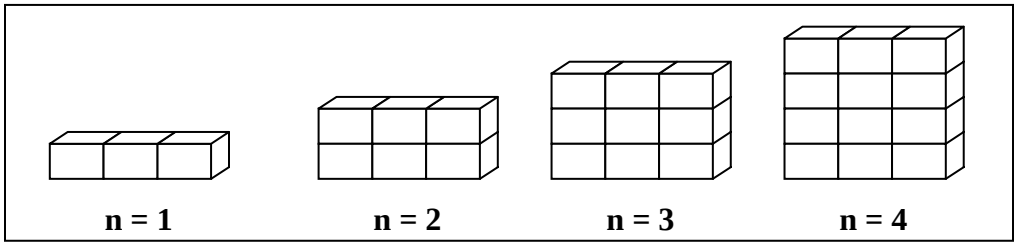
Home Activity or Further Classroom Consolidation

Reflect on the activity in your math journal. What patterns did you discover? What would you change if you were to do it again?

25.1: Cubey Doobey Doo

Name:
Date:

Complete all of your work in your notebook, with the exception of question 1.



1. Complete the following tables for the objects above.

Volume (V)	
n	V
1	
2	
3	
4	
5	

Surface Area (A)	
n	A
1	
2	
3	
4	
5	

Edge Length (E)	
n	E
1	
2	
3	
4	
5	

2. Describe each of the number patterns for V, A, and E in words.

3. Find the surface area for the object whose figure number is 15. Use the units for area. Explain how you got your answer.

4. Find the figure number if the volume is 72 cubic units. Explain how you got your answer.

5. Is an edge length of 67 units possible in this pattern? If not, find the closest edge length that is possible.
   Explain how you got your answer.

6. On orthographic dot paper, draw the front, top, and right side orthographic views for $n = 2$. On isometric dot paper draw the left and right isometric views for $n = 2$. Use a ruler to make your drawing.

7. Draw the net for the object with figure number $n = 2$ and then use it to build the object.
   Hint:
   Trace the faces of your object onto a piece of paper in the appropriate configuration. Use a ruler. Include flaps for gluing your object.

Source “Linking Assessment and Instruction: Middle Years” with permission from OAME

25.1: Cubey Doobey Doo (continued)
(Answers)

1. Complete the following tables for the objects above. Continue the number sequences on the table where needed.

Volume (V)	
n	V
1	3
2	6
3	9
4	12
5	15

Surface Area (A)	
n	A
1	14
2	22
3	30
4	38
5	46

Edge Length (E)	
n	E
1	20
2	24
3	28
4	32
5	36

2. Volume = 3 multiplied by *n*

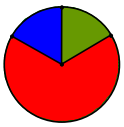
   Surface Area = 8 multiplied by *n,* plus 6.
   The surface area increases by 8 each time *n* increases by 1.
   Using 8×1 for $n = 1$ generates $14 - 8 = 6$ as a value to be added to each multiple of 8.

   Edge Length = 4 multiplied by *n,* plus 16.
   The edge length increases by 4 each time *n* increases by 1.
   Using 4×1 for $n = 1$ generates $20 - 4 = 16$ as a value to be added to each multiple of 4.

3. Students may use a variety of methods to solve this, including actual creation of the model, guess and check using their description of the patterns or extending the table to $n = 15$ to see that surface area = 126 units^2. Discuss the advantages and disadvantages of each.

4. A variety of methods may again be used.
   If V = 72, $n = 24$

5. Edge length must be even, so 67 units in not possible. The closest possible length is 68 units, since 68 – 16 = 52 and 52 is divisible by 4. Students will use a variety of methods to solve this.



Description

- Build rectangular prisms using manipulatives.
- Develop and apply the formula for finding the volume of a rectangular prism.

Materials

- linking cubes
- isometric dot paper
- orthographic dot paper·
- BLM 25.1

Assessment Opportunities

Minds On ...

Whole Class → Review

Show students a column of 4 cubes. What is the volume of this rectangular prism? Attach another column of 4 cubes beside with the first one. (This makes a prism 4 units high, 2 units wide, and 1 unit deep.) What is its volume? Add a second 2 × 4 prism to the original (forms a prism that is 4 units high, 2 units wide and 2 units deep). Students predict the volume of the prism and explain their thinking.

Action!

Pairs → Model Making

Curriculum Expectations/Observation/Rubric: Assess students' ability to make an appropriate conjecture about the formula for volume, based on the data they have collected. Students should be able to provide reasonable justification of their conjecture.

Students build three different rectangular prisms, using linking cubes and sketch the prisms on isometric paper. The length of each side is equal to one unit.

They create a table to show the relationship between the base area and height of a rectangular prism to its volume and develop a formula for the volume of a rectangular prism.

Formulas may be expressed using words or symbols or algebraically.

V = area of the base × height of the prism

Connection: A sleeve of crackers or a package of paper provides a visual image of an area (1 cracker/1 sheet) multiplied by height.

Consolidate Debrief

Whole Class → Student Presentation

Students share their formulas for calculating volume of a rectangular prism. Reinforce the concept of cubic units.

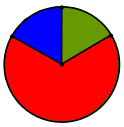
Select a student to add vocabulary to the Word Wall.

Home Activity or Further Classroom Consolidation

Application
Concept Practice

Take necessary measurements to find surface area and volume for an empty cereal box or other package. Record diagrams and calculations. When calculating surface area do not include any overlaps or tabs on the ends.

Complete the following questions for additional practice: (The teacher inserts text references.)



Description

- Design a cereal box that meets certain specifications (a fixed volume).
- Apply volume and surface area formulas for rectangular prisms.
- Present designs to the class.

Materials

- cereal boxes
- isometric dot paper
- orthographic dot paper

Assessment Opportunities

Minds On ...

Whole Class → Discussion

Display several cereal boxes from the class collection. Tell the students that a cereal manufacturer has hired them to create a box for a new cereal.

Discuss the parameters of the project, Cereal Box Challenge (BLM 27.1). Note that the company has set forth certain restrictions concerning the volume and surface area of the box.

Explain that in pairs they are to determine the dimensions of a box that will meet the requirements on the challenge, but that the package design will be an individual activity.

Students may wish to discuss overlaps of box board to form the box.

See TIP 6 for suggestions on how to link this activity with other curriculum.

Action!

Individual → Performance Task

Curriculum Expectations & Learning Skills/Performance Task/Checklist & Rubric: Assess students' work using a rubric.

Students use dot paper to investigate possible dimensions that meet the manufacturer's criteria. They should find at least two possible solutions before they select the dimensions that they will use for their box.

Consolidate Debrief

Whole Class → Student Presentations

Curriculum Expectations/Observation/Checklist: Some pairs sketch their box on the board and explain the process they used to determine the dimensions of their box.

What is the same with the various processes used? How do they differ?

Application
Concept Practice

Home Activity or Further Classroom Consolidation

In your math journal, explain the steps used to determine appropriate dimensions for the cereal box. Include nets and a 3-D sketch of the box.

27.1: Cereal Box Challenge

Cereal Box Challenge

You work for an advertising company that as been asked to submit a proposal for the design of a box for a new breakfast cereal. Through its research, the manufacturer of the cereal has determined the optimum volume of the cereal box and a range of surface areas to minimize the packaging costs.

Your job is to:

- design and construct a box to meet the following criteria:
  - volume of 12 000 cm^3
  - surface area of the box between 3200 cm^2 and 4000 cm^2
  - the box has a stable base and pleasing proportions

- design packaging with shelf appeal, including:
  - a unique name for a cereal that would appeal to 10 – 18 year-olds
  - placement of consumer information
  - appropriate graphics
  - colour scheme

Submit the following for assessment:

- the sketches you used to determine the dimensions of your proposed cereal box
- all calculations used to determine the dimensions and surface area
- rough copies of your package designs
- the final mock-up of your cereal box (actual size)

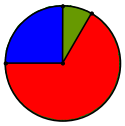
27.2: Assessment Tool Cereal Box Design

Did the student meet basic criteria for design?

Volume is 12 000 cm^3	Yes	No
Surface Area is between 3200 cm^2 and 4000 cm^2	Yes	No
Design is stable	Yes	No

Mathematical Process (Category)	Criteria	Below Level 1	Level 1	Level 2	Level 3	Level 4
Reasoning and Proving (Problem Solving) volume/surface area	Evidence of self-monitoring (revisions to dimensions)	- no evidence	- limited evidence	- some evidence	- evidence	- evidence of attention to fine detail
Communicating (Communication) design concepts	Clarity (explanations and presentations) Use of conventions (accurately, effectively, fluently)	- unclearly - demonstrates an undeveloped use of conventions	- with limited clarity - demonstrates minimal skill in the use of conventions	- with some clarity - demonstrates moderate skill in the use of conventions	- clearly - demonstrates considerable skill in the use of conventions	- precisely - demonstrates a high degree of skill in the use of conventions

Learning Skills	Needs Improvement	Satisfactory	Good	Excellent
Independent Work				
• follows routines and instructions without supervision				
• persists with tasks				
Initiative				
• responds to challenges				
• demonstrates positive attitude towards learning				
• develops original ideas and innovative procedures				
• seeks assistance when necessary				
Use of Information				
• organizes information logically and creatively and manages it effectively				
• asks questions to clarify meaning and ensure understanding				



Description

- Design a cereal box that meets certain specifications (a fixed volume).
- Apply volume and surface area formulas for rectangular prisms.
- Present their designs to the class.

Materials

- cereal boxes
- construction supplies: Bristol board, markers, etc.

Assessment Opportunities

Minds On ...

Pairs → Conferencing

Students meet to share their final package designs and to make any suggestions for changes in design and/or use of colour.

Action!

Individual → Model Making

Students use Bristol board to create the net for their cereal box, using the dimensions determined on Day 27.

Students' transfer their designs to Bristol board before they construct the box.

Students may need several days to create the cereal box.

Consolidate Debrief

Whole Class → Student Presentations

Students present their final product to the class. Provide some guidance concerning their presentation "what to present," e.g., actual model; measurements and resulting volume, surface area; highlight one design decision and provide reasons.

Alternatively, a math fair could be set up displaying all designs with a written response.

Home Activity or Further Classroom Consolidation

Both the following sequences show growth patterns:

Sequence A: 1, 2, 4, 8, 16, …

Sequence B: 2, 4, 6, 8, 10, …

Discuss similarities and differences between these sequences.

Exploration

The next unit targets exponents. This activity is meant to help students re-orient their thinking, activating prior knowledge from lessons on Days 5 to 8.

Lesson Outline: Days 29 - 31

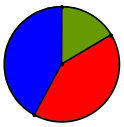
Grade 7

BIG PICTURE

Students will:

- understand that exponents are symbols to illustrate repeated multiplication of the same base number;
- discover the power of exponential growth;
- investigate patterns.

Day	Lesson Title	Description	Expectations
29	Some Friendly Advice	• Investigate exponential growth to develop an understanding of exponential notation as repeated multiplication.	7m4, 7m12 CGE 2c, 3c
30	Payday!	• Investigate a pattern to develop understanding of a power. Use a scientific calculator to evaluate powers.	7m71, 7m72 CGE 2c, 4a
31	Penny Panic!	• Use standard and non-standard units to estimate the volume of containers.	7m28, 7m44 CGE 5a



Description

- Investigate exponential growth to develop an understanding of exponential notation as repeated multiplication.

Materials

- BLM 29.1, 29.2

Assessment Opportunities

Minds On ...

Whole Class/Read Along

Read BLM 29.1 with the students. Individually, students spend a few minutes responding to Rani's father's request for advice. Discuss their responses. Ask students to predict the amount Rani will receive on Day 12. Encourage students to express their prediction in a variety of ways, e.g., about 300, more than 3 thousand, between 600 and 700, 80. Record the students' predictions for discussion later in the task.

This scenario acts as an effective illustration of the amazing power of doubling.

Action!

Pairs → Activity

Students select the month that they wish to complete the data for and fill in BLM 29.2 to the 12^{th} day following the pattern established in the story (1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1 024, 2 048), using calculators as required.

Introduce the notion that each of the numbers in the chart can be expressed as products of repeated factors, e.g., $8 = 2 \times 2 \times 2$ where 8 is the product of three 2s.

It is important to focus the students on prime factoring ($8 = 2 \times 2 \times 2$)

Review with students the relationship of repeated factors to exponential notation (e.g., $8 = 2 \times 2 \times 2 = 2^3$), noting the patterns, e.g., the exponent "3" matches the number of repeated factors.

Exponents can be a complex idea for some students. For example, some students may not really understand that 3 squared (3^2) is not the same as 3×2.

Students record each number for the first 12 days in the chart as repeated factors and then using exponents. Students reflect on the usefulness of the exponent notation.

Consolidate Debrief

Pairs → Reflection

Individually or in pairs, students identify any patterns that they see in the different notations and should provide justification for patterns that they identify.

Curriculum Expectations/Observation/Mental Note: Assess correct use of mathematical language and clarity of explanation.

Whole Class → Debrief

Facilitate a class discussion and record student observations on the overhead chart or on the board.

Observations may include:

- The exponent is the same as the number of repeated factors, e.g., $16 = 2 \times 2 \times 2 \times 2 = 2^4$.
- The exponent is one less than the number of the day, e.g., Day 6 so $2^5 = 32$.

Connect exponents as they relate to measurement of squares and cubes that students have already encountered.

Area of square: $4 \times 4 = 16$ square units (or 4^2)

Volume of cube: $4 \times 4 \times 4 = 64$ cubic units (or 4^3)

Home Activity or Further Classroom Consolidation

Respond in your math journal: Writing the symbols for exponents is more efficient than listing multiple factors. Support your opinion with examples.

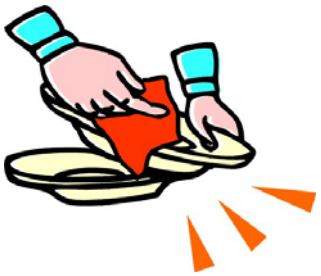
Select questions for practice in writing numbers in exponential form and as the product of multiple factors.

Reflection
Skill Drill

Do you agree with your initial advice for Rani's father? Explain your thinking in your math journal. Predict how much money Rani would earn on the 31^{st} day if she accepted the second option. Explain how you made your prediction.

Complete the questions from your text: (Teacher selects exercises)

29.1: Doing the Dishes



Name:
Date:

Rani's father is tired of constantly reminding Rani to wash the dishes every day. He considers offering Rani the following options:

Option 1:
$1 for each day

Option 2:
1¢ on the first day, 2¢ on the second day, 4¢ on the third day, and doubling each day.

He is considering having Rani select one payment option which they will try for one month.

Rani's father has asked for some advice.

Are these appropriate options to offer Rani?
Explain your reasoning.

29.2: _____ Chart

Name:
Date:

Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**	**Day 7**
Day 8	**Day 9**	**Day 10**	**Day 11**	**Day 12**	**Day 13**	**Day 14**
Day 15	**Day 16**	**Day 17**	**Day 18**	**Day 19**	**Day 20**	**Day 21**
Day 22	**Day 23**	**Day 24**	**Day 25**	**Day 26**	**Day 27**	**Day 28**
Day 29	**Day 30**	**Day 31**				

Description

- Investigate a pattern to develop understanding of a power. Use a scientific calculator to evaluate powers.

Materials

- calculators
- BLM 29.2

Assessment Opportunities

Minds On ...

Small Group → Discussion

Students share their journal responses and contribute to a class discussion. Ensure that all students have an understanding of exponential notation. Note students' predictions and processes for prediction.

The exponent is one less than the number of the day, e.g., day 5 yields 2^4 because you started with 2^0

Discuss why this strategy may be advantageous to recursive estimates (doubling every day until day 30.)

Action!

Small Group → Activity

Assign each group a portion of the remaining days (13 to 31). Each group calculates and records how many pennies Rani would receive for each of their assigned days. They express the amount of pennies for each day using exponential notation.

Curriculum Expectations/Observation/Feedback: Depending on the method they chose, groups may need assistance with calculator use (see sidebar). Students are expected to use calculators efficiently.

Differentiated Instruction

Students who are still working with recursive estimates should be assigned the days closer to day 12 so they can gain confidence in working with the concept of exponents. Students who have demonstrated a comfort level with exponents can complete the days closer to 31.

Note that on some scientific calculators the answer to the last few days may be displayed using scientific notation. Students may need assistance correctly interpreting this form.

Whole Class → Check for Accuracy

Share the results of the group calculations, and students check and complete their charts. They compare the answer for day 31 with their estimate from their math journal.

Consolidate Debrief

Whole Class → Debrief

As a class, discuss the patterns on the chart, adding to the list produced during the previous lesson. Challenge students to develop a procedure to determine *n*th term, such as the 35^{th} term. They will have to use the generalization that the exponent is one less than the number of the day.

Challenge students to find a reasonable strategy for determining the total number of pennies over the 31-day period. Discuss possible solutions.

Lead students to the realization that although Rani's father may think that the second option would be reasonable, in fact, the power of exponential growth makes this an unrealistic option.

Home Activity or Further Consolidation

Concept Practice

Write a situation using a different exponential pattern and provide some mathematical questions with solutions. Choose a situation which generates powers of 3, 4 or some other number.

For practice of expanded and exponential notation, complete these questions, using a calculator: (textbook or worksheet questions)

Note: The repeat or doubling feature may be different on various calculators. Check this before using with the class.

For scientific calculators, students may need help with using the exponent key.

If simple 4-function calculators are available, demonstrate the doubling feature, e.g., Enter a number, press × key, then repeatedly press = key to reach the desired exponent.

Description

- Use standard and non-standard units to estimate the volume of containers.

Materials

- BLM 29.2 Chart
- calculators
- round counters
- various sizes of containers

Assessment Opportunities

Minds On ...

Pairs → Activity

Students exchange questions they developed about exponential patterns. Each student reads, reviews, and solves the problem posed by their partner. Students revise their work based on the feedback and experience of their partners.

Curriculum Expectations/Demonstration/Rubric: Observe students' skills as they solve the problem posed. Assessment can focus on problem solving skills, use of appropriate language and clarity of explanation.

For more information on Fermi problems, refer to:

- Impact Math: *Number Sense, Numeration*. Ontario Ministry of Education, 1999 (pages 16-18).
- Linking Assessment and Instruction in *Mathematics: Middle Years.* OAME/OMCA, 2001. (Fun With Fermi, pages 116-135)
- TIP 3 Fermi Problems

Action!

Think/Pair/Share → Investigation

Suppose Rani's father offered both options and Rani accepted the second option. She would have to decide what to do with all the pennies. Today's task is to figure out the type and number of containers that would be necessary to hold the amount of pennies Rani would be paid on various days.

Students individually record a process that they will use to solve this problem, e.g., a margarine container holds 100 counters. After talking with a partner they revise their strategy, as appropriate.

Students work in pairs to solve the problem, referring to the particular container and process they chose. Select specific days for each pair. Each pair of students uses the data to predict how many containers will be required to hold all of the pennies.

Consolidate Debrief

Whole Class/Discussion

Record answers on the chart (BLM 29.2). Ask pairs to share the types and quantities of containers they visualized for day 31.

Discuss the pattern evident in the growing number of containers and determine why the pattern makes sense in relation to the other patterns.

In small groups discuss: What was the most challenging part of this assignment? What strategies did you use to accomplish the task? What do you think the phrases "...it goes up exponentially, or exponential growth" mean?

The groups share the information with the class.

When returning graded work to students, consider photocopying samples of Level 3 and 4 responses with student names removed. Select and discuss, with the class, samples that show a variety of strategies.

Application

Home Activity or Further Classroom Consolidation

Write a report, a story, or a skit using the idea of exponential growth.

Grade	7
Total time	180 minutes
Materials	• Data on disk, *Corel Quattro-Pro*, *Microsoft Excel* or *Appleworks*
Description	Students use technology to investigate problem solving strategies in dealing with the Olympic medals, the GDP, and population and area of a country to determine a country's success at the Olympics.
Expectations **Assessed* and addressed**	**Number Sense and Numeration** 7m24 – *explain the process used and any conclusions reached in problem solving and investigations; 7m25 – reflect on learning experiences and describe their understanding using appropriate mathematical language; 7m26 – solve problems involving fractions and decimals using the appropriate strategies and calculation methods. **Patterning and Algebra** 7m69 • apply and discuss patterning strategies in problem-solving situations; 7m70 – describe patterns in a variety of sequences using the appropriate language and supporting materials; 7m75 – present solutions to patterning problems and explain the thinking behind the solution process. **Data Management and Probability** 7m83 – *use computer applications to examine and interpret data in a variety of ways; 7m85 – *construct graphic organizers using computer applications; 7m86 • interpret displays of data and present the information using mathematical terms; 7m87 – *evaluate data and make conclusions from the analysis of data; 7m93 – understand the difference between a spreadsheet and a database for recording and retrieving information; 7m94 – search databases for information and interpret the numerical data; 7m98 – use conventional symbols, titles, and labels when displaying data; 7m102 – display data on bar graphs, pictographs, and circle graphs, with and without the help of technology; 7m104 – *evaluate arguments that are based on data analysis; 7m105 – *explore with technology to find the best presentation of data. **Ontario Catholic School Graduate Expectations** CGE 3c – thinks reflectively and creatively to evaluate situations and solve problems; CGE 5a – works effectively as an interdependent team member.
Prior Knowledge	Working knowledge of a spreadsheet program: • how to use the sort feature • how to enter/use simple formulas • how to generate a variety of graphs
Assessment Tools	Rubric
Extensions	What formula would weight a country's total medals per GDP at 3 points and its total medals per person in the country at 5 points? Use this formula to determine the top three countries. Discuss merits of this formula.

Pre-task Instructions

Read the following script to students on the day before the investigation begins.

Teacher Script

1. An investigation is an extended problem designed to allow you to show your ability to undertake an inquiry: to make a hypothesis, formulate a plan, collect data, model and interpret the data, draw conclusions, and communicate and reflect on what you have found.

2. We will be working on an investigation over the next three mathematics classes.

3. For this investigation you will be using a spreadsheet. As well you will need pencils, pens, an eraser, a ruler, notepaper and a calculator to complete the work.

4. As you do the investigation you will work as part of a group and also individually. (*Distribute an envelope or folder to each studen*t.) I am giving each of you an envelope in which you can store your notes for the duration of the investigation. Write your name on the front of the envelope.

5. On the third day you will write a summary giving your conclusions and summarizing the processes you have followed to arrive at them.

6. Be sure to show your work and include as much explanation as needed.

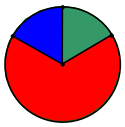
7. Each section of the investigation has a recommended time limit that I will tell you so you can manage your time.

8. You will be assigned to the following groups for the three days of the Investigation. (*You may wish to assign students to their groups at this time – recommended group size is four students*.)

9. Are there any questions you have regarding the format or the administration of the investigation?

Teacher Notes

- This summative task could be used for gathering summative assessment data or for providing formative feedback to students before they complete another task for assessment purposes.
- If a Home Activity or Further Classroom Consolidation task is to be used for gathering assessment data, it may be most appropriate for students to work on it independently under teacher supervision.
- Some suggested Home Activity or Further Classroom Consolidation tasks help prepare students for later assessments.



Description

- Understand the complexity of the problem.
- Work by hand with a large amount of data in order to appreciate the need for spreadsheets.

Materials

- BLM S1.1

Minds On ...

Whole Class → Guided Discussion

Read the problem. (BLM S1.1)

Explain to the students about the International Olympic Committee (IOC) policy regarding medal tables.

Discuss: Why do the media publish tables of medal standings?

Possible answers:

- to show how successful a country has been
- to see which country is the "best"
- to see how well a country's Olympic organization measures up

Gross Domestic Product (GDP) is the market value of all the goods and services produced in a nation's economy during a given period.

Action!

Small Groups → Brainstorm

Students use the Olympic data to determine how they would rank the top 3 countries.

Whole Class → Sharing

Each of the groups presents their ideas on how they would rank the top 3 countries, given the information in the table.

Small Groups → Worksheet

Assign each member 1 or 2 questions from question 1 – 6 (BLM S1.1) to answer without using technology. They share their findings. Students determine if they still agree with their original choice for the top three countries.

Since there is more work involved in question 2, you may wish to assign more than one student to this question.

Consolidate Debrief

Whole Class → Sharing

Ask groups to share their answers to questions 1 – 6 with the class.
In answer to question 7, lead a discussion on how different point values, population, area, and GDP might be used in determining a country's standing. Area is the factor least likely to be considered in determining a country's standing. There may be a relationship between:

- population and a country's standing
- GDP and a country's standing

Changing the point values of different medals may influence a country's standing.

Application
Concept Practice
Exploration
Reflection

Home Activity or Further Classroom Consolidation

Answer in your journal:
Give 2 different reasons why it is more impressive that Cuba won 11 gold medals than that Britain won 11 gold medals.

S1.1 Introducing the Problem

In the Olympics, medals are awarded for placing first, second, or third in an event. The first place winner gets the gold medal, the second place winner gets the silver medal, and the third place winner gets the bronze medal. The International Olympic Committee (IOC) does not approve of keeping track of medals that countries have won. However, tables that report the "success" of different countries are frequently used in the popular media. The table below was taken from an unofficial website. It shows the list of countries who participated in the 2002 Summer Olympics, along with the number of gold, silver, and bronze medals that were awarded.

Task:
You will conduct investigations and write a summary that describes how a country can determine its success at the Olympics.

Country Medal Winnings:
Sydney, 2000

Country	Gold	Silver	Bronze	Total	GDP per capita	Population	Area
Algeria	1	1	3	5	$ 5 500	31 133 486	2 381 740
Argentina	0	2	2	4	$12 900	36 737 664	2 736 690
Armenia	0	0	1	1	$ 3 000	3 409 234	28 400
Australia	16	25	17	58	$23 200	18 783 551	7 617 930
Austria	2	1	0	3	$25 000	8 139 299	82 738
Azerbaijan	2	0	1	3	$ 3 000	7 908 224	86 100
Bahamas	1	1	0	2	$15 000	283 705	10 070
Barbados	0	0	1	1	$14 500	259 191	430
Belarus	3	3	11	17	$ 7 500	10 401 784	207 600
Belgium	0	2	3	5	$25 300	10 182 034	30 230
Brazil	0	6	6	12	$ 6 500	171 853 126	8 456 510
Britain	11	10	7	28	$22 800	59 113 439	241 590
Bulgaria	5	6	2	13	$ 6 200	8 194 772	110 550
Cameroon	1	0	0	1	$ 1 700	15 456 092	469 440
Canada	3	3	8	14	$24 800	31 006 347	9 220 970
Chile	0	0	1	1	$10 100	14 973 843	748 800
China	28	16	15	59	$ 3 600	1 246 871 951	9 326 410
Colombia	1	0	0	1	$ 6 200	39 309 422	1 038 700
Costa Rica	0	0	2	2	$ 6 700	3 674 490	50 660
Croatia	1	0	1	2	$ 5 800	4 676 865	56 410
Cuba	11	11	7	29	$ 1 700	11 096 395	110 860
Czech Republic	2	3	3	8	$12 900	10 280 513	78 645
Denmark	2	3	1	6	$25 500	5 356 845	42 394
Estonia	1	0	2	3	$10 000	1 408 523	43 211
Ethiopia	4	1	3	8	$600	59 680 383	1 119 683
Finland	2	1	1	4	$22 900	5 158 372	305 470
France	13	14	11	38	$24 400	58 978 172	545 630
Georgia	0	0	6	6	$ 4 600	5 066 499	69 700
Germany	13	17	26	56	$23 400	82 087 361	349 520
Greece	4	6	3	13	$17 200	10 707 135	130 800
Hungary	8	6	3	17	$11 200	10 186 372	92 340
Iceland	0	0	1	1	$24 800	272 512	100 250
India	0	0	1	1	$ 2 200	1 000 848 550	2 973 190
Indonesia	1	3	2	6	$ 2 900	216 108 345	1 826 440
Iran	3	0	1	4	$ 6 300	65 179 752	1 636 000.04

Country Medal Winnings:
Sydney, 2000

Country	Gold	Silver	Bronze	Total	GDP per capita	Population	Area
Ireland	0	1	0	1	$21 600	3 632 944	68 890
Israel	0	0	1	1	$18 900	5 749 760	20 330
Italy	14	8	13	35	$22 100	56 735 130	294 020
Jamaica	0	4	3	7	$ 3 700	2 652 443	10 830
Japan	5	8	5	18	$24 900	126 182 077	374 744
Kazakhstan	3	4	0	7	$ 5 000	16 824 825	2 669 800
Kenya	2	3	2	7	$ 1 500	28 808 658	569 250
Kuwait	0	0	1	1	$15 000	1 991 115	17 820
Kyrgyzstan	0	0	1	1	$ 2 700	4 546 055	191 300
Latvia	1	1	1	3	$ 7 200	2 353 874	64 589
Lithuania	2	0	3	5	$ 7 300	3 584 966	65 200
Mexico	1	2	3	6	$ 9 100	100 294 036	1 923 040
Moldova	0	1	1	2	$ 2 500	4 460 838	33 371
Morocco	0	1	4	5	$ 3 500	29 661 636	446 300
Mozambique	1	0	0	1	$ 1 000	19 124 335	784 090
Netherlands	12	9	4	25	$24 400	15 807 641	33 889
New Zealand	1	0	3	4	$17 700	3 662 265	268 670
Nigeria	0	3	0	3	$950	113 828 587	910 770
Norway	4	3	3	10	$27 700	4 438 547	307 860
Poland	6	5	3	14	$ 8 500	38 608 929	304 510
Portugal	0	0	2	2	$15 800	9 918 040	91 951
Romania	11	6	9	26	$ 5 900	22 334 312	230 340
Russia	32	28	28	88	$ 7 700	146 393 569	16 995 800
Saudi Arabia	0	1	1	2	$10 500	21 504 613	1 960 582
Slovakia	1	3	1	5	$10 200	5 396 193	48 800
Slovenia	2	0	0	2	$12 000	1 970 570	20 256
South Africa	0	2	3	5	$ 8 500	43 426 386	1 219 912
Spain	3	3	5	11	$18 000	39 167 744	499 400
Sweden	4	5	3	12	$22 200	8 911 296	410 928
Switzerland	1	6	2	9	$28 600	7 275 467	39 770
Thailand	1	0	2	3	$ 6 700	60 609 046	511 770
Turkey	3	0	3	6	$ 6 800	65 599 206	770 760
Ukraine	3	10	10	23	$ 3 850	49 811 174	603 700
United States	40	24	33	97	$36 200	272 639 608	9 158 960
Uzbekistan	1	1	2	4	$ 2 400	24 102 473	425 400
Yugoslavia	1	1	1	3			

http://sydney2000.espn.go.com/medaltracker/totalmedal?sort=total&page=2
http://www.photius.com/wfb1999/rankings/population_1html

In Your Group

1. If you rank the countries by number of Gold medals won, what are the top three countries?

2. Suppose you award 10 points for a Gold, 5 points for a Silver, and 1 point for a Bronze. What are the top three countries?
   How about 3 points for a Gold, 2 points for a Silver, 1 point for a Bronze?

3. Which three countries have the most people?

4. Which three countries are the smallest by area?

5. Which three countries have the highest GDP?

6. Which three countries have the lowest GDP?

7. What factors do you think should be considered in determining how well a country has done at the Olympics?

Description

- Use a spreadsheet to manipulate and analyse a large amount of data.

Materials

- BLM S1.2
- spreadsheet software

Minds On ...

Whole Class → Guided Discussion

Pose the question Why is it easier to manipulate a data set like the 'Olympic Medals' using a spreadsheet rather than by hand?

Give the students the disc with the data base.

The countries can quickly be sorted according to various categories, it is easy to apply a ranking system to the entire set. Formulas can be applied to all of the data at once.

Action!

Pairs → Explore

Students use the spreadsheet to sort the data to answer the questions on the BLM S1.2.

Small Group → Planning

Students identify which criteria is the most important in helping determine the ranking of the countries.

Using that criteria, students must create a formula to manipulate the data.

Pairs → Investigate

Students manipulate the data using the formula created in the group to determine the ranking of the countries.

Encourage pairs to explore other options for looking at the data — ways in which they can graphically look at comparing the data. For example, they can use a double bar graph to represent the performance of the three smallest countries compared to the three largest countries.

Transfer the Olympic data to a spreadsheet program for the students to use.

Consolidate Debrief

Small Group → Discussion

The students, in turn, share any discovery they made while they were manipulating the data.

Home Activity or Further Classroom Consolidation

Exploration Reflection

Answer the following question in your journal:

- How do you feel about the fact that some of the mathematics problems you have to solve do not have just one correct answer?
- What makes problems like Olympic Data so challenging?
- Why do you suppose the curriculum requires that you solve problems like this?

S1.2 Exploring Patterns

In partners:

Using the sorting command on the spreadsheet answer the following questions:

1. If you rank the countries by Gold, what are the top 3 countries?

2. Which three countries have the most people?

3. Which three countries are the smallest by area?

4. Which three countries have the highest GDP?

5. Which three countries have the lowest GDP?

Create a column to enter a formula to answer the following question:
Suppose you award 10 points for a Gold, 5 points for a Silver, and 1 point for a Bronze. What are the top 3 countries?

In your group:

Determine which criteria is the most important in determining the ranking of the countries.

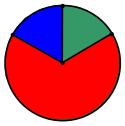
Write a formula that can be used to incorporate the criteria in ranking the countries.

In partners:

Use the spreadsheet to manipulate the data using the formula that you created.

Explore other options such as:

- 3 points for Gold, 2 for Silver and 1 for Bronze
- creating a double bar graph to represent the performance of the three smallest countries to the three largest countries



Description

- select and apply a problem-solving strategy.
- justify choice of strategy.
- use a spreadsheet to manipulate and present data.

Materials

- BLM S1.3
- spreadsheet software

Minds On ...

Whole Class → Guided Discussion

Each of the groups shares with the class the criteria they used to rank the countries.

Action!

Individual → Report Writing

Students independently write a summary that describes how a country can determine its success at the Olympics (BLM S1.3). Their summary should include:

- their ranking system
- advantages and disadvantages of their ranking system
- assumptions they made
- graphical analysis

Consolidate Debrief

Whole Class → Reflection

Compare the ranking of the countries with the original ranking from Day 1 done by hand. Are the choices the same? Comment on why or why not.

Home Activity or Further Classroom Consolidation

Identify which curriculum expectations apply to your method of solving the Olympic Medalling problem.

OR

What formula would weight a country's total medals per GDP at 3 points vs. its total medals per person in the country at 5 points? Use this formula to determine the top 3 countries. Discuss merits of this formula.

Application
Reflection
Differentiated

Curriculum expectations could be photocopied for students to use as a checklist and study guide

S1.3 More Exploring and Conclusions

Write a summary that describes how a country can determine its success at the Olympics. The summary should include:

- organized data;
- strategies used;
- interpretation of data;
- your ranking system recommendations with justification;
- assumptions you have made;
- advantages and disadvantages of your ranking system;
- graphical analysis.

S1.4 Sample Solutions for Teachers

As sorted data shows, the top three countries ranked by Gold are United States, Russia and China.

Using the point allocation criteria, apply the formula

10 × # of Gold + 5 × # of Silver + 1 × # of Bronze

or

3 × # of Gold + 2 × # of Silver + 1 × # of Bronze

and sort the obtained data. The result in both cases puts, again, United States, Russia and China in the first three spots.

Sorting population data we determined that the three countries that have the most people are China, India and United States. Russia is in the sixth place.

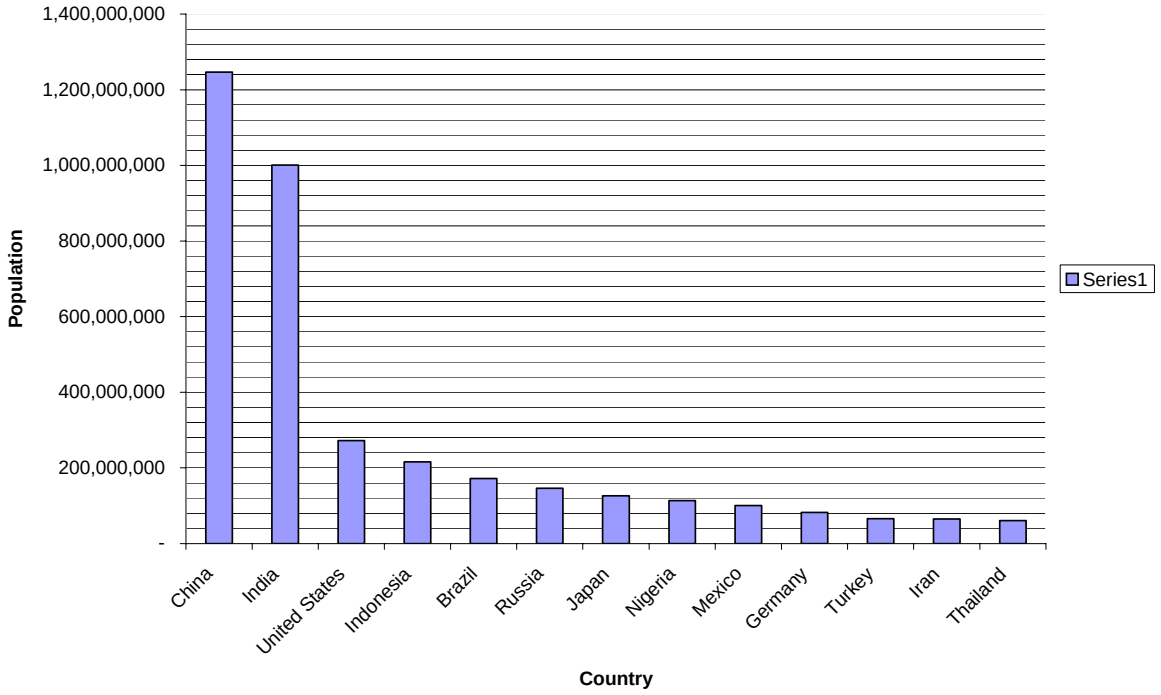
The smallest countries by area are Barbados, Bahamas, and Jamaica. These countries are ranked middle to low in the medals' allocation.

The richest countries were determined by sorting them based on the GDP data. According to this criteria the richest countries are United States, Switzerland, and Norway. Although United States ranks first in the medals' allocation, the other two "richest" countries rank towards the middle.

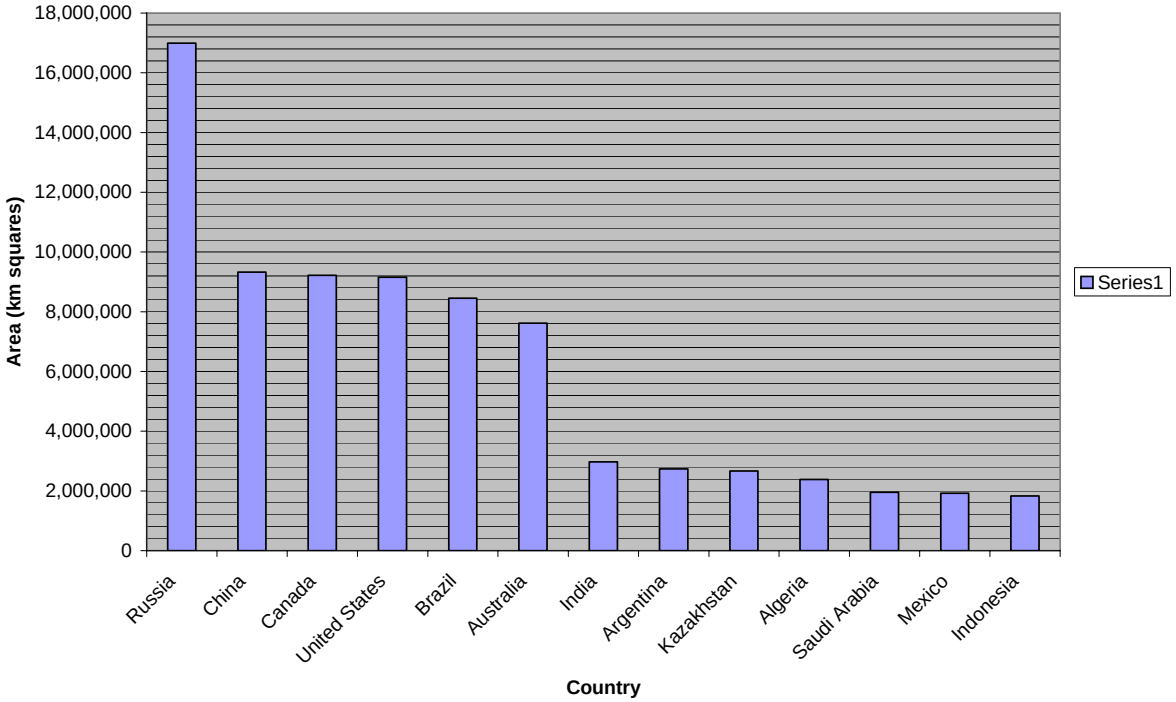
Conclusion

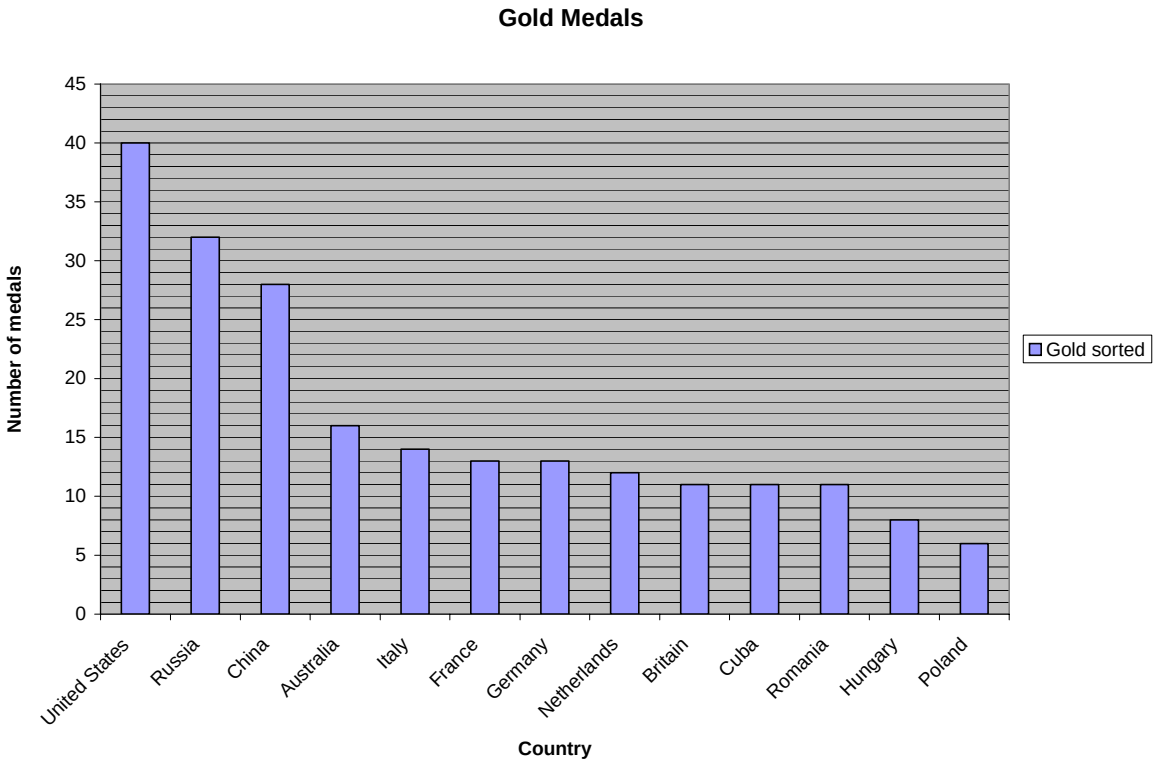
The top three medal winners are United States, Russia and China. These three countries have large populations, are big in size but are quite different on the rich-poor scale.
There is a correlation between the population of a country, its size and the number of medals. The more people a country has, the easier is to find talented, competitive and hard working people. Also there is a slight correlation between the richness of a country and the number of medals as rich countries can sponsor and allocate money to encourage participation in sports. However, Russia and China, according to their GDP are ranked middle to low and are top medal winners.

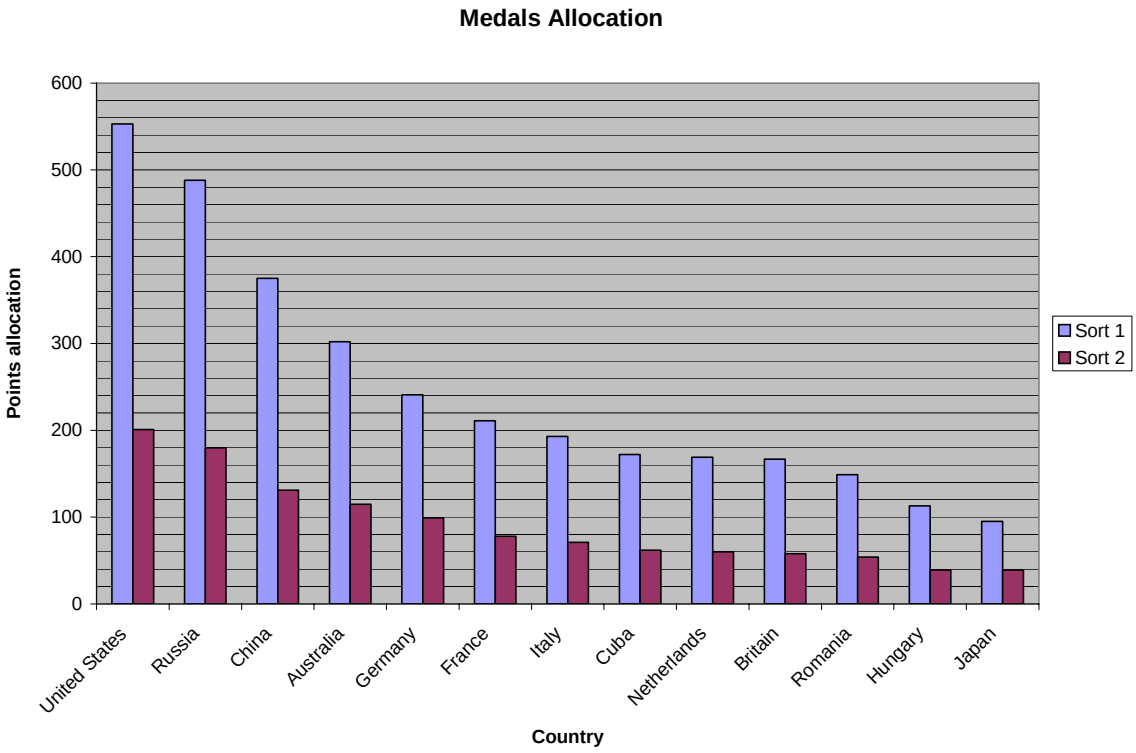
Population Chart



Area Chart







Assessment Tool: Olympic Data

Reasoning and Communication can be assessed through observation during the investigation as well as through the student's written submission.

Criteria	Below L1	Level 1	Level 2	Level 3	Level 4
Making Connections (Understanding of Concepts)					
Collects and organizes suitable data	- little or no evidence	- with major omissions or errors	- with some omissions of errors	- completely	- completely includes evidence of reflection
Develops effective algebraic and graphical models	- little or no evidence	- narrow fit to situation	- moderate fit to situation	- broad fit to situation	- very extensive fit to situation
Reasoning and Proving (Problem Solving)					
Makes convincing argument, explanations and justification for ranking system	- no evidence of logic - no conclusions reached	- limited logic evident - major omission in arriving at conclusion	- somewhat logical - some omission in arriving at conclusions	- logical - thorough	- highly logical - complete and extended
Communicating (Communication)					
Uses mathematical language	- undeveloped use of conventions	- minimal skill in use of conventions	- moderate skill in use of conventions	- considerable skill in use of conventions	- high degree of skill in use of conventions
Explains the processes and conclusions	- unclear/ confusing	- limited clarity	- some clarity	- clear	- precise
Knowing Facts and Procedures (Application)					
Calculations and Computations	Assess for correctness using a marking scheme				

Grade	7
Total time	180 minutes
Materials	• *The Geometer's Sketchpad*® (dynamic geometry software) • Object with at least one square face, rulers or tape measures, chart paper, BLMs
Description	Using manipulatives and technology students collect data to investigate the relationship between the perimeter and the length of the longest diagonal of a regular polygon (with an even number of sides). Students discover a pattern and use it to solve a problem. Students discover that in an even-sided regular polygon, the ratio of the perimeter to the length of the longest diagonal is a constant. They submit a report that justifies and explains their conclusions.
Expectations **Assessed* and addressed**	**Number Sense and Numeration** 7m5 • use estimation to justify or assess the reasonableness of calculations; 7m7 • explain, in writing, the process of problem solving using appropriate mathematical language; 7m23 – ask "what if" questions; pose problems involving simple fractions, decimals, and percents; and investigate solutions; 7m24 – *explain the process used and any conclusions reached in problem solving and investigations; 7m25 – reflect on learning experiences and describe their understanding using appropriate mathematical language (e.g., in a math journal); 7m26 – solve problems involving fractions and decimals using the appropriate strategies and calculation methods. **Measurement** 7m28 • demonstrate a verbal and written understanding of and ability to apply accurate measurement strategies that relate to their environment; 7m33 – describe measurement concepts using appropriate measurement vocabulary; 7m35 – *****make increasingly more informed and accurate measurement estimations based on an understanding of formulas and the results of investigations. **Geometry and Spatial Sense** 7m52 • use mathematical language effectively to describe geometric concepts, reasoning, and investigations; 7m62 – create and analyse designs that include translated, rotated, and reflected two-dimensional images using concrete materials and drawings, and using appropriate computer applications. **Patterning and Algebra** 7m67 • identify, extend, create, and discuss patterns using whole numbers and variables; 7m69 – *apply and discuss patterning strategies in problem-solving situations; 7m70 – describe patterns in a variety of sequences using the appropriate language and supporting materials; 7m71 – *extend a pattern, complete a table, and write words to explain the pattern; 7m72 – *recognize patterns and use them to make predictions; 7m75 – present solutions to patterning problems and explain the thinking behind the solution process.

Expectations **Assessed* and addressed**	**Data Management and Probability** 7m81 – *systematically collect, organize, and analyse data; 7m83 • use computer applications to examine and interpret data in a variety of ways; 7m84 • develop an appreciation for statistical methods as powerful means of decision making; 7m85 • construct graphic organizers using computer applications; 7m87 • evaluate data and make conclusions from the analysis of data; 7m95 – understand that each measure of central tendency (mean, median, mode) gives different information about the data; 7m98 – use conventional symbols, titles, and labels when displaying data; 7m101 – read and report information about data presented on bar graphs, pictographs, and circle graphs, and use the information to solve problems; 7m103 – *make inferences and convincing arguments that are based on data analysis. **Ontario Catholic School Graduate Expectations** CGE 3c – thinks reflectively and creatively to evaluate situations and solve problems; CGE 5a – works effectively as an interdependent team member.
Prior Knowledge	Skills with *The Geometer's Sketchpad*® Students should be able to: • select/deselect one or more objects • construct points and line segments • perform a "drag test" to see if a sketch holds its shape
Assessment Tools	Rubric
Extensions	This investigation considers only regular polygons with an even number of sides. What would be the same and what would be different if you used polygons with an odd number of sides?

Pre-task Instructions

Read the following script to students on the day before the investigation begins. Note that students need to bring an object for Day 1 of the investigation – see #10 below.

Teacher Script

1. We will be working on an investigation over the next three mathematics classes.

2. An investigation is an extended problem designed to allow you to show your ability to undertake an inquiry: to make a hypothesis, formulate a plan, collect data, model and interpret the data, draw conclusions, and communicate and reflect on what you have found.

3. For this investigation you will be using *The Geometer's Sketchpad*®. As well, you will need pencils, pens, an eraser, a ruler, notepaper, and a calculator to complete the work.

4. As you do the investigation you will work as part of a group and also individually. (*Distribute an envelope or folder to each student*.) I am giving each of you an envelope in which you can store your notes for the duration of the investigation. Write your name on the front of the envelope.

5. On the third day you will write a report giving your conclusions and summarizing the processes you have followed to arrive at them.

6. Be sure to show your work and include as much explanation as needed.

7. Each section of the investigation has a recommended time limit that I will tell you so you can manage your time.

8. You will be assigned to the following groups for the three days of the investigation. (*You may wish to assign students to their groups at this time – recommended group size is four students*.)

9. Are there any questions you have regarding the format or the administration of the investigation?

10. On the first day of the investigation you will need to bring an object that has at least one square face.

Teacher Notes

- This summative task could be used for gathering summative assessment data or for providing formative feedback to students before they complete another task for assessment purposes.
- If a Home Activity or Further Classroom Consolidation task is to be used for gathering assessment data, it may be most appropriate for students to work on it independently under teacher supervision.
- Some suggested Home Activity or Further Classroom Consolidation tasks help prepare students for later assessments.

Description

- Gather data to determine the ratio perimeter: length of diagonal for a square without using technology.
- Apply this ratio to solve a problem.

Materials

- Object with at least one square face
- rulers or tape measures
- BLM S2.1, S2.2

Minds On ...

Whole Class → Guided

Read BLM S2.1 to ensure that students understand the problem.

What is a gazebo?

Why is it better to purchase the trim immediately?

Action!

Small Groups → Collecting Data

Students follow instructions on BLM S2.1 to collect measurements for their square. Each student enters group data on the chart.

Identify objects in the classroom that have at least one square face or cut some squares from paper for students who may not have their own object.

Small Groups → Discussion

Groups discuss patterns/relationships in the table. (#3, BLM S2.1.)

The ratio of the perimeter to the length of the diagonal is 2.83 (rounded to two decimal places) for any square! (S2.2 Answers)

Results may vary because of errors in measurements or calculations.

Four rows might not be sufficient data for students to see the relationship.

Small Groups → Collecting Data

Students exchange data with 7 other students so that each student eventually has 8 rows of data.

What precision does your measuring device have?

Small Groups → Discussion

Students return to groups after collecting the data and discuss:

- Did the additional data support your hypothesis? Give reasons for your answer.
- Do you need more data? Give reasons for your answer.

Consolidate Debrief

Whole Class → Guided Discussion

Discuss student responses, e.g., I think that for any square the ratio of the perimeter to the length of the diagonal is 2.83.

Individual data pieces that are very different within a set are called outliers. What causes outliers? What should you do with them?

Did students eliminate outliers? Did they use the mean (median or mode)?

Discuss the questions and give examples:

If a blob of ink fell onto your worksheet and covered …

a) … the length of side column, how would you determine the missing number?
b) … the length of diagonal column, how would you determine the missing number?
c) … the ratio column, how would you determine the missing number?
d) … the perimeter column, how would you determine the missing number?
e) … the perimeter and the length of side column, how would you determine the missing numbers?

Why aren't there any units for the ratio?

Individual → Assessment

Give students 5 minutes to complete the following assessment question.

How much trim will be needed if your family decides to make a square-based gazebo with a diagonal of length 4.0 m? Describe the method you used and show your work. Collect student work.

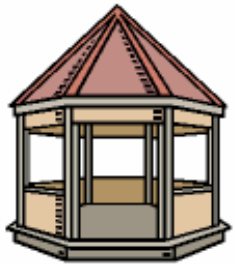
Home Activity or Further Classroom Consolidation

Complete the following questions to prepare for tomorrow.

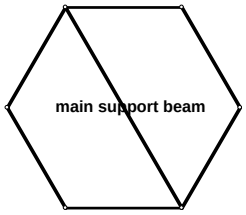
Application
Skill Drill

Assign textbook questions that review properties and construction of hexagons and octagons.

S2.1 Introducing the Problem



Your class is going to build a gazebo for the local seniors' home. The gazebo's base will be a regular polygon. You have enough room in the seniors' yard to make the main support beam 4.0 metres long. Your gazebo will have trim along the perimeter of the base. Trim is on sale but you are not ready to make a final decision for the shape of the base of your gazebo. Investigate a variety of designs to determine the amount of trim you need to purchase at the sale price.



The main floor support beam (4.0 metres) will be the longest diagonal in the regular polygon.

Start with a simpler problem – investigate a square-based gazebo

1. Take measurements for any square to complete the first row of the table.
2. Enter data from other group members into the empty rows.
3. Discuss in your group any patterns or relationships you see in the chart.
4. Make a hypothesis by completing the statement: I think …

	Length of Side (cm)	Perimeter (cm)	Length of Diagonal (cm)	$\frac{\text{Perimeter}}{\text{Length of Diagonal}}$
1				
2				
3				
4				
5				
6				
7				
8				

S2.2 Determining Ratio: Math Note

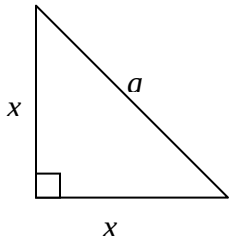
Answers

The following algebraic solution to determine the ratio of the perimeter to the diagonal is beyond the scope of the Grade 7 curriculum. It is included as information for the teacher only.

The actual value for the ratio can be found as follows:

Using the Pythagorean theorem in a right-angled triangle, we determine that:

$$x^2 + x^2 = a^2$$
$$2x^2 = a^2$$
$$x^2 = 0.5a^2$$
$$\sqrt{x^2} = \sqrt{0.5a^2}$$
$$x \doteq 0.707a$$



Perimeter ÷ Diameter
$= 4x \div a$
$\doteq 4\ (0.707a) \div a$
$= 2.828\ a \div a$
$= 2.828$

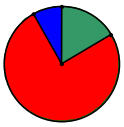
Since the a's divide out in the last step, we know that the ratio does not depend on the value of *a*. The ratio is 2.828 for any square!

Students will discover some of these ratios:

Regular Polygon	Ratio
Square	2.83
Hexagon	3.00
Octagon	3.06
Decagon	3.09
30-sided polygon	3.14

With many-sided polygons, students discover that the ratio becomes about 3.14, which is an approximate value of π.

Note: Knowing that their result is an approximation of π is not expected in Grade 7. This is a Grade 8 expectation.



Description

- Gather data to determine the ratio of perimeter: length of diagonal for a regular hexagon using GSP.
- Apply this ratio to solve a problem.
- Investigate the ratio for regular octagons by hand.

Materials

- *The Geometer's Sketchpad®* Version 4
- GSP file: The Gazebo
- BLM S2.3, S2.4, S2.5, S2.6

Minds On ...

Whole Class → Guided

Discuss solutions to problem from Day 1.

Part of Possible Solution:

Perimeter/Diagonal = 2.83

Diagonal = 4.0 m

Perimeter = 4.0 x 2.83

= 11.32 m

Some students may use a scale diagram rather than the ratio to determine the perimeter.

If a student was unsuccessful with the question submitted from Day 1, provide assistance or arrange brief peer tutoring.

Action!

Think/Pair/Share → Conferencing

Students complete questions 1 and 2 on BLM S2.3 and share responses in pairs then with the whole group.

Pairs → Investigation

Students use *The Geometer's Sketchpad®* to determine that the ratio of the perimeter to the length of the longest diagonal in a regular hexagon is also a constant.

Time for this activity will vary depending on students' familiarity with *The Geometer's Sketchpad®* software. If your students are not familiar with using GSP you may wish to give them one of the additional instruction sheets for constructing regular hexagons, e.g., BLM S2.4 or BLM S2.5, or use the pre-made sketches shown on S2.6.

Establish/review classroom practices for:

- saving electronic files of student work, e.g., Katie2.gsp identifies the second file that belongs to Katie
- printing, e.g., students include a text box with their name and date

When printing a GSP file, first select Print Preview from the File menu, and then Scale to fit page

Consolidate Debrief

Individual → Assessment

Give students 5 minutes to complete the following assessment question.

How much trim would be needed if your class decided to make the base of the gazebo, a regular hexagon with a diagonal of length 4.0 m? Use the data you collected. Describe the method you use and show your work. Collect student work.

Application
Concept Practice
Exploration
Skill Drill

Home Activity or Further Classroom Consolidation

Construct 3 different-sized regular octagons. Use a ruler to measure side lengths and the longest diagonal. Calculate perimeter/diagonal.

S2.3 Exploring Patterns

1. In any square, (length of perimeter): (length of diagonal) = _____

2. Do you think this ratio will be the same for a regular hexagon? Give reasons for your answer.

3. Investigate your hypothesis by using *The Geometer's Sketchpad®.*

   1) Construct:
      - a regular hexagon
      - the longest diagonal

   2) Measure:
      - length of one side of the hexagon
      - length of the longest diagonal

   3) Calculate:
      - the perimeter
      - the ratio of the perimeter of the hexagon to the length of the diagonal

   Record the data, then collect more entries for the table using GSP.

Length of Side (cm)	Perimeter (cm)	Length of Diagonal (cm)	Perimeter: Diagonal

4. If a blob of ink fell onto your table and covered the perimeter and the length of side, how would you determine the missing numbers?

GSP V4 Instructions

Constructing a Hexagon by Rotating Diagonals

Set Distance Units

- Under the Edit menu choose Preferences.
- Set distance units to cm.
- Set precision to hundredths.

Saving Files

- Establish classroom procedures.

Printing Sketches

- Use the text tool to create a text box with your name and date.
- Under the File menu choose Print Preview – then select Scale to Fit Page – then choose Print.

Construct a Hexagon

1. Construct a line segment.
2. Under Construct choose Point at Midpoint.
3. Under Transform choose Mark Center.
4. Select the line segment and its endpoints.
5. Under Transform choose Rotate.
6. Enter 60o for the angle of rotation.
7. Under Transform choose Rotate.
8. Deselect, then choose the segment tool to construct the sides of the hexagon.
9. Do a drag test.

Take Measurements

1. Select the two endpoints of one side of the hexagon.
2. Under Measure choose Distance. (The measurement appears in the sketch.)
3. Select the two endpoints of one diagonal of the hexagon.
4. Under Measure choose Distance.

Make Calculations

<u>Calculate the Perimeter</u>

1. Under Measure choose Calculate.
2. Select 6, select *, select the measurement of the side length, then press OK.

<u>Calculate the Ratio of Perimeter to Diagonal</u>

1. Under Measure choose Calculate.
2. Select the Perimeter measurement, select ÷, select the measurement of the side length, then press OK.
3. Do a drag test and watch the measurements change.

Create a Table

1. Select all of the following measurements and calculations: side length, diagonal length, perimeter, ratio.
2. Under Graph choose Tabulate.

New Table Entry

1. Change the side length of your hexagon by dragging a point in the diagram.
2. Place the arrow on one of the table entries.
3. Double click and a new entry will be added.

   Repeat steps 1-3 to add more entries.

GSP V4 Instructions

Constructing a Hexagon by Rotating the Sides

Set Distance Units

- Under the Edit menu choose Preferences.
- Set distance units to cm.
- Set precision to hundredths.

Saving Files

- Establish classroom procedures.

Printing Sketches

- Use the text tool to create a text box with your name and date.
- Under the File menu choose Print Preview – then select Scale to Fit Page – then choose Print.

Construct a Hexagon

1. Construct a line segment.
2. Select the one endpoint.
3. Under Transform choose Mark Center.
4. Select the line segment and its endpoints.
5. Under Transform choose Rotate.
6. Enter 120o for the angle of rotation. Press Rotate.
7. Mark the new endpoint created as the center of rotation. (Double click the endpoint.)
8. Repeat steps 4 to 7 to create a hexagon.
9. Do a drag test.

Take Measurements

1. Select the two endpoints of one side of the hexagon.
2. Under Measure choose Distance. (The measurement appears in the sketch.)
3. Select the two endpoints of one diagonal of the hexagon.
4. Under Measure choose Distance.

Make Calculations

Calculate the Perimeter

1. Under Measure choose Calculate.
2. Select 6, select *, select the measurement of the side length, then press OK.

Calculate the Ratio of Perimeter to Diagonal

1. Under Measure choose Calculate.
2. Select the Perimeter measurement, select ÷, select the measurement of the side length, then press OK..
3. Do a drag test and watch the measurements change.

Create a Table

1. Select all of the following measurements and calculations: side length, diagonal length, perimeter, ratio.
2. Under Graph choose Tabulate.

New Table Entry

1. Change the side length of your hexagon by dragging a point in the diagram.
2. Place the arrow on one of the table entries.
3. Double click and a new entry will be added.

   Repeat steps 1-3 to add more entries.

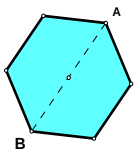
The Gazebo (GSP file)

Download this file at www.curriculum.org/occ/tips/downloads.shtml

Investigating Regular Hexagons

Explore:
Drag point A and notice which measurements change.

Collect Evidence:
To add a new entry to the table, drag point A then double click in the table. Add several rows to the table.



Investigate a Regular Octagon

Measurements

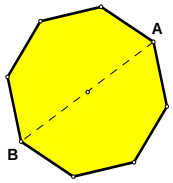
Length of one side = 1.98 cm
Perimeter = 11.88 cm
Length of Diagonal = 3.96 cm
$\frac{\text{Perimeter}}{\text{(Length of Diagonal)}} = 3.00$

Length of Diagonal	Perimeter	$\frac{\text{Perimeter}}{\text{(Length of Diagonal)}}$
3.96 cm	11.88 cm	3.00

Investigating Regular Octagons

Explore:
Drag point A and notice which measurements change.

Collect Evidence:
To add a new entry to the table, drag point A then double click in the table. Add several rows to the table.



Investigate a Regular Decagon

Measurements

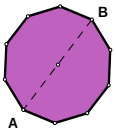
Length of one side = 1.95 cm
Perimeter = 15.57 cm
Length of Diagonal FG = 5.09 cm
$\frac{\text{Perimeter}}{\text{(Length of Diagonal FG)}} = 3.06$

Length of Diagonal FG	Perimeter	$\frac{\text{Perimeter}}{\text{(Length of Diagonal FG)}}$
5.09 cm	15.57 cm	3.06

Investigating Regular Decagons

Explore:
Drag point A and notice which measurements change.

Collect Evidence:
To add a new entry to the table, drag point A then double click in the table. Add several rows to the table.



Investigate 30-sided Polygons

Measurements

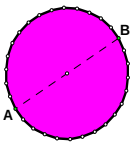
Length of one side = 1.16 cm
Perimeter = 11.57 cm
Length of Diagonal = 3.74 cm
$\frac{\text{Perimeter}}{\text{(Length of Diagonal)}} = 3.09$

Length of Diagonal	Perimeter	$\frac{\text{Perimeter}}{\text{(Length of Diagonal)}}$
3.74 cm	11.57 cm	3.09

Investigating Regular Polygons with 30 Sides

Explore:
Drag point A and notice which measurements change.

Collect Evidence:
To add a new entry to the table, drag point A then double click in the table. Add several rows to the table.



Explore regular polygons with an odd number of sides

Measurements

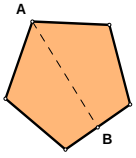
Length of One Side = 0.44 cm
Perimeter = 13.21 cm
Length of Diagonal = 4.21 cm
$\frac{\text{Perimeter}}{\text{(Length of Diagonal)}} = 3.14$

Length of Diagonal	Perimeter	$\frac{\text{Perimeter}}{\text{(Length of Diagonal)}}$
4.21 cm	13.21 cm	3.14

Investigating Regular Pentagons

Explore:
Drag point A and notice which measurements change.

Collect Evidence:
To add a new entry to the table, drag point A then double click in the table. Add several rows to the table.



Explore regular polygons with a larger odd number of sides.ide

Measurements

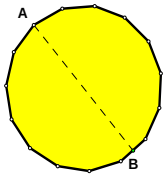
Length of one side = 2.66 cm
Perimeter = 13.30 cm
Line of Symmetry = 4.09 cm
$\frac{\text{Perimeter}}{\text{(Line of Symmetry)}} = 3.25$

Line of Symmetry	Perimeter	$\frac{\text{Perimeter}}{\text{(Line of Symmetry)}}$
4.09 cm	13.30 cm	3.25

Investigating Regular Polygons with 15 Sides

Explore:
Drag point A and notice which measurements change.

Collect Evidence:
To add a new entry to the table, drag point A then double click in the table. Add several rows to the table.

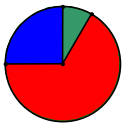


Return to Beginning

Measurements

Length of One Side = 1.11 cm
Perimeter = 16.69 cm
Line of Symmetry = 5.29 cm
$\frac{\text{Perimeter}}{\text{(Line of Symmetry)}} = 3.15$

Perimeter	Line of Symmetry	$\frac{\text{Perimeter}}{\text{(Line of Symmetry)}}$
16.69 cm	5.29 cm	3.15



Description

- Use GSP to investigate perimeter: length of diagonal for regular octagons.
- Solve the original problem.

Materials

- *The Geometer's Sketchpad®*
- BLM S2.7

Minds On ...

Whole Class → Guided

Discuss solutions to problem from Day 2.

Students might use the length of the main beam of the gazebo (4.0 m) and the ratio they found in the previous activity to determine the amount of trim.

Perimeter/Diagonal = 3.00

Diagonal = 4.0 m

Perimeter = 4.0 x 3.00

= 12.0 m

Students should discover that the more sides the polygon has, the closer the ratio of perimeter to diagonal is 3.14. Ideally, if one does not know exactly what type of polygon base the gazebo is then one could use this approximation to establish an estimated amount of trim needed to purchase.

Action!

Independent → Investigation

Students use *The Geometer's Sketchpad®* to determine that the ratio of the perimeter to the length of the longest diagonal in a regular octagon is also a constant (BLM S2.7). Time for this activity will vary depending on students' familiarity with *The Geometer's Sketchpad®*.

Encourage students to explore other even-sided polygons using *The Geometer's Sketchpad®* to see the relationship of the perimeter to the diagonal.

Consolidate Debrief

Individual → Assessment

Students complete a report of their findings (15 minutes). The report should include a solution to the problem (BLM S2.7), and a conclusion on how they can determine the amount of trim needed for the base of any even-sided regular polygon gazebo. See Assessment Tool: The Gazebo.

Application
Concept Practice
Reflection
Skill Drill

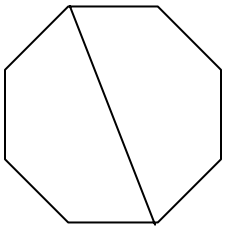
Home Activity or Further Classroom Consolidation

To be determined by teacher.

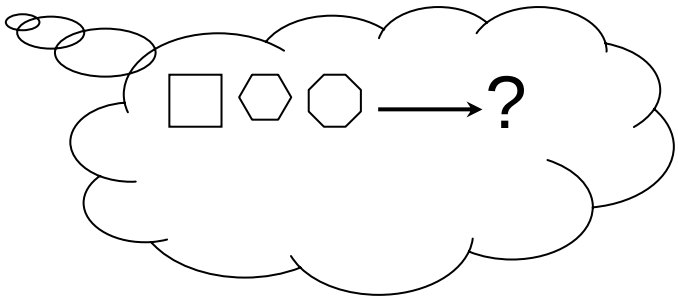
Suggestion: Reflective journal entry.

More Exploring and Conclusions

1. How much trim would be needed if your class decided to make the base of the gazebo a regular octagon? Describe the method you use and show your work.



2. You have examined a number of options for the base of the gazebo. Investigate other even-sided regular polygons to help you determine the amount of trim you will purchase at the sale price. Record your observations and notes.



3. Write a summary with the details about what you have done and the conclusions you have come to about the original problem.

   - State a specific answer for the amount of trim you would purchase.
   - Justify your conclusions – explain how you determined your answer.
   - Include references to any notes, sketches, or tables you used.
   - Suggest any additional steps you would follow if you had more time.

Your class is going to build a gazebo for the local seniors' home. The gazebo's base will be a regular polygon. You have enough room in the seniors' yard to make the main support beam 4.0 metres long. Your gazebo will have trim along the perimeter of the base. Trim is on sale but you are not ready to make a final decision for the shape of the base of your gazebo. Investigate a variety of designs to determine the amount of trim you need to purchase at the sale price.

Assessment Tool: The Gazebo

Criteria	Below L1	Level 1	Level 2	Level 3	Level 4
Making Connections (Understanding of Concepts)					
Extends a pattern	- little or no evidence	- with major omissions or errors	- with some omissions or errors	- completely	- completely and includes evidence of reflection or verification
Collects suitable data and organizes data	- little or no evidence	-with major omissions or errors	- with some omissions or errors	- completely	- completely and includes evidence of reflection or verification
Is able to generalize pattern	- little or no evidence	- narrow fit to situation	- moderate fit to situation	- broad fit to situation	- very extensive fit to situation
Reasoning and Proving (Problem Solving)					
Makes convincing argument, explanations and justifications	- no evidence of logic	- limited logic evident	- somewhat logical	- logical	- highly logical
	- no conclusions reached	- major omissions in arriving at conclusion	- some omissions arriving at conclusion	- thorough	- complete and extended
Communicating (Communications)					
Uses mathematical language	- undeveloped use of conventions	- minimal skill in use of conventions	- moderate skill in use of conventions	- considerable skill in use of conventions	- high degree of skill in use of conventions
Explains the processes and conclusions	- unclear/ confusing	- limited clarity	- some clarity	- clear	- precise
Knowing Facts and Procedures (Application)					
Calculation and computation	Assess for correctness using a marking scheme				

