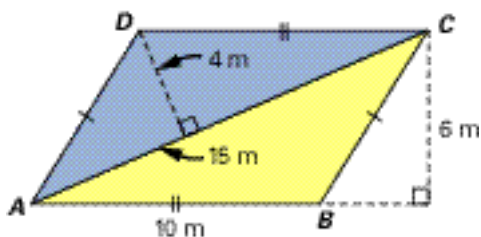# 5.2 Area of a Triangle

▶ **GOAL**

Develop and apply the formula for the area of a triangle.

Learn about the Math

Chang and Sandra are calculating the cost of painting the floor of an outdoor stage. The measurements are shown in the diagram below. The price of the paint is $2.25 per square metre.

❓ What is the cost of painting the yellow half of the stage?

Sandra says, "We know how to calculate the area of a parallelogram. Can we use the area of the parallelogram to calculate the area of the yellow triangle?"

Chang says, "The area of the parallelogram is $b \times h = 10 \text{ m} \times 6 \text{ m}$, or 60 m^2. If I cut the parallelogram along line segment AC, I can match the two triangles. The area of the yellow triangle is half the area of the parallelogram. So, the area of $\triangle ABC = 60 \text{ m}^2 \div 2$, or 30 m^2."


Communication Tip

• $\triangle ABC$ is a short way to refer to a triangle that has vertices A, B, and C. $\triangle ABC$ is read "triangle $A B C$."

• In diagrams, identical marks are used to show that line segments are the same length. Sides AD and BC are the same length. Sides AB and DC are the same length.

Sandra says, "The cost of painting 1 m^2 is $2.25. So, the cost of painting 30 m^2 is $30 \times \$2.25$, or $67.50."

Example 1: Calculating the area of a triangle by forming a parallelogram

How can you calculate the area of △ABC by forming a parallelogram?

Chang's Solution
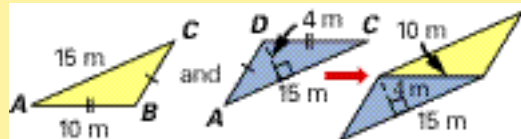
I matched sides *AB* and *DC* by sliding the blue triangle underneath the yellow one. This formed a parallelogram. The base is 15 m. The height is 4 m.

$A = b \times h$
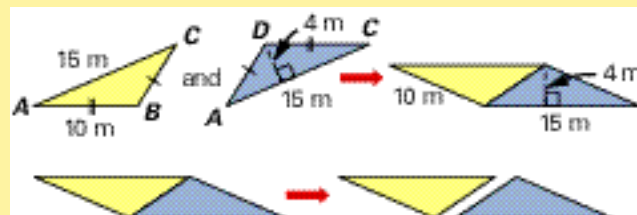$= 15 \text{ m} \times 4 \text{ m}$
$= 60 \text{ m}^2$

I multiplied to find the area of this parallelogram.

The two triangles split the parallelogram in half. So, the area of one triangle is half the area of the parallelogram.

$A = 60 \text{ m}^2 \div 2$
$= 30 \text{ m}^2$

The area of △ABC is 30 m².

Sandra's Solution

I matched sides *CB* and *DA* to form a different parallelogram than Chang's. The base is 15 m, and the height is 4 m.

My calculations would be the same as Chang's, so the area of △ABC is 30 m².
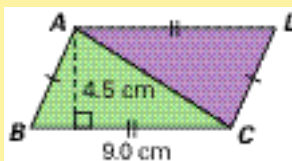
Reflecting

1. Why can every triangle be thought of as half a parallelogram?

2. What is the formula for the area of a triangle in terms of the base (*b*) and height (*h*) of the related parallelogram?

Work with the Math

Example 2: Calculating the area of a triangle

Calculate the area of △ABC to the nearest tenth.

Solution

$A = (b \times h) \div 2$
$= (9.0 \text{ cm} \times 4.5 \text{ cm}) \div 2$
$= 40.5 \text{ cm}^2 \div 2$
$= 20.25 \text{ cm}^2$

To find the area of a triangle, multiply the base (*b*) by the height (*h*), and divide by 2.

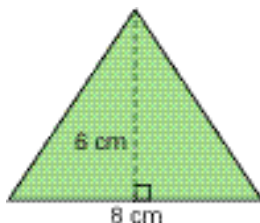△ABC has a base of 9.0 cm and a height of 4.5 cm.

The area of △ABC, rounded to the nearest tenth, is 20.3 cm².

Ⓐ Checking

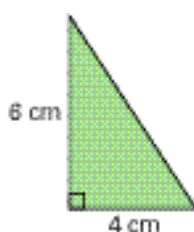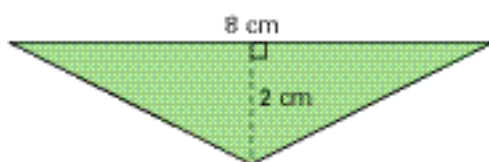**3.** Show how each triangle is half a parallelogram. Then calculate the area of the triangle.
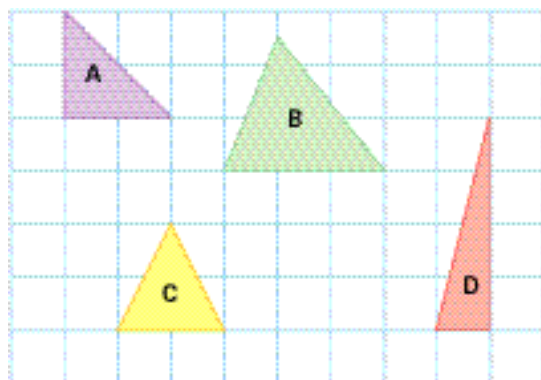
a)

6 cm
8 cm

b)

6 cm
4 cm

c)

8 cm
2 cm

4. On centimetre grid paper, draw two triangles with each area.

   a) 12 cm^2 **c)** 10 cm^2

   b) 18 cm^2 **d)** 8 cm^2

5. Draw the following triangles on centimetre grid paper, and estimate the area of each triangle. Based on your estimates, predict whether any of the triangles have the same area. Check by measuring and calculating.
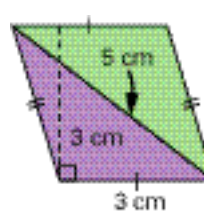
Ⓑ Practising
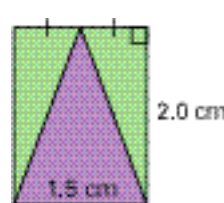
6. Copy and complete the table.

	Base	Height	Area of triangle
a)	4 cm	9 cm	▧ cm^2
b)	12 cm	45 cm	▧ cm^2
c)	3.5 mm	6.0 mm	▧ mm^2
d)	6.0 m	7.5 m	▧ m^2

7. Calculate the area of each purple triangle.

a)

5 cm
3 cm
3 cm

b)

2.0 cm
1.5 cm

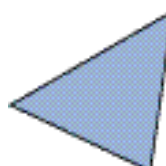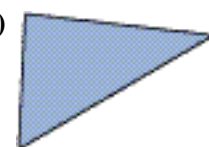**8.** Measure a base and a height for each triangle. Then determine each area.

a)   **c)**

b)   **d)**

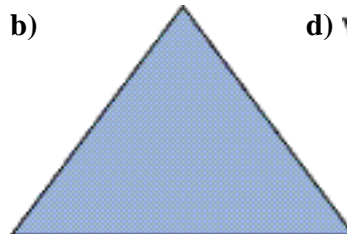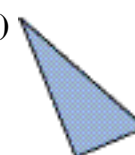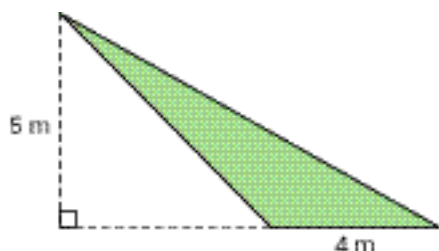**9.** △*ABC* has three different pairs of bases and corresponding heights, as shown.

   a) Does every triangle have three different pairs of bases and corresponding heights? Explain.

   b) When you use a formula to calculate the area of a triangle, does it matter which height-base pair you use? Explain.
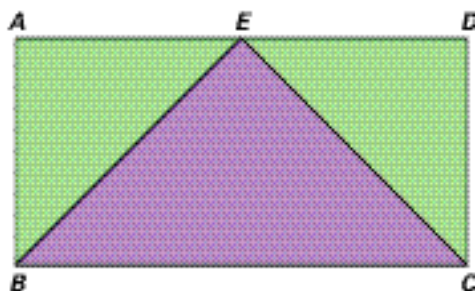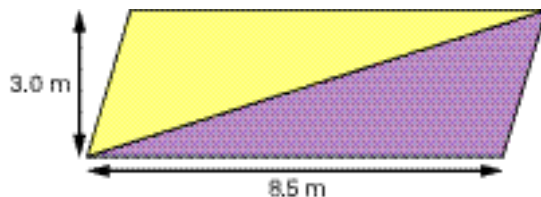
10. Calculate the area of the triangle.

5 m

4 m

11. Copy and complete the table.

	Base	Height	Area of triangle
a)	6 cm	■ cm	72 cm^2
b)	■ m	6.0 m	10.2 m^2
c)	40 mm	9 cm	■ cm^2
d)	250 mm	■ cm	625 cm^2

12. Calculate the area of each shape by measuring the sides.

A   E   D

B   C

   a) $\triangle BEC$ **c)** $\triangle ABE$

   b) $\triangle EDC$ **d)** rectangle *ABCD*

13. a) Calculate the area of the yellow fabric in the flag.
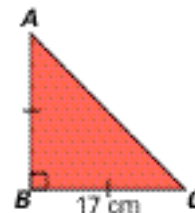

3.0 m

8.5 m

   b) The price of the yellow fabric is $8.40/m^2. Calculate the cost of the yellow fabric for one flag.

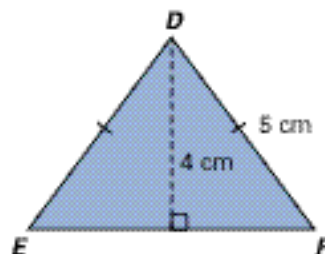   **c)** Calculate the area of yellow fabric that is needed for 30 flags.

Ⓖ Extending

14. The perimeter of $\triangle ABC$ is 58 cm.

   a) How long is *AB*?

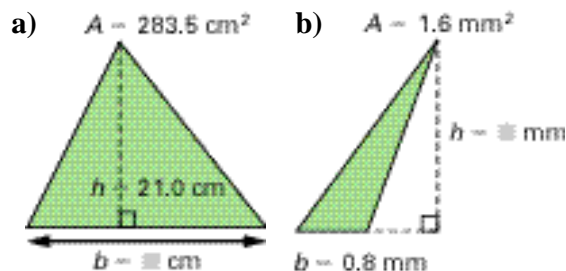   **b)** Calculate the area of $\triangle ABC$.

A

B   17 cm   C

15. The perimeter of $\triangle DEF$ is 16 cm.

   a) How long is *EF*?

   b) Calculate the area of $\triangle DEF$.

D

5 cm

4 cm

E   F

16. Determine each missing value.

   a) $A \approx 283.5$ cm^2 **b)** $A \approx 1.6$ mm^2

$h \approx 21.0$ cm

$b \approx$ ■ cm

$h \approx$ ■ mm

$b \approx 0.8$ mm

17. Design a kite by combining four triangles. Calculate the area of each triangle and the total area of the kite.