Solving Equations by Inspection

▶ **GOAL**

Write equations and solve them by inspection.

Learn about the Math

John and Heidi are building this toothpick pattern. They have 28 toothpicks.

Figure 1   Figure 2   Figure 3

? **How many squares can they build for this pattern with 28 toothpicks?**

A. Copy and complete the first four rows of the table of values for the toothpick pattern. Write the pattern rule for the relationship between the number of squares and the number of toothpicks.

Number of squares	Number of toothpicks
1	4
2	
3	
4	

B. Use the variable s to represent the number of squares. Write an algebraic expression that describes the relationship in step A.

C. Complete the **equation** $t = \blacksquare$, where t represents the number of toothpicks you need, and $\blacksquare$ is your algebraic expression from step B.

D. When the number of toothpicks is 28, the value of t in your equation is 28. Rewrite your equation with 28 substituted for t.

E. Part of your equation should be $\blacksquare s$. Explain how you can figure out the value of $\blacksquare s$ in your equation. Then explain how you can figure out the value of s.

F. Check your **solution to the equation** by building the squares.

Reflecting

1. Why would continuing the table of values not be an efficient strategy for solving this problem?

equation

a mathematical statement in which the value on the left side of the equal sign is the same as the value on the right side of the equal sign; for example, $6d + 1 = 13$ is an equation for 6 times d plus 1 equals 13

solution to an equation

the value of a variable in an equation that makes the equation true

2. The method you used to solve the equation in step E is called "solving by inspection." The word "inspect" means "examine carefully." Explain how your solution strategy used a careful examination of the equation.

Work with the Math

Example 1: Solving an equation by inspection

Use the method of inspection to solve the equation $2a - 1 = 13$.

Kaitlyn's Solution

$2a - 1 = 13$
$2a = 14$
$a = 7$

I looked at the equation.
Since you subtract 1 from $2a$ to get 13, $2a$ must be 14.
Since 2 times a number is 14, the number must be 7.

Check.
Left side   Right side
$2a - 1$ 13
$= 2(7) - 1$
$= 13$ ✓

I checked my solution by substituting it into the equation to see if it works.
My solution $a = 7$ works, so it is correct.

Example 2: Solving a problem by solving an equation

The cost to rent skates is $3 plus $2 per hour. Omar has $21. What is the greatest number of hours he can rent the skates?

Omar's Solution

My equation for the rental cost is

$c = 3 + 2h$
$21 = 3 + 2h$

$18 = 2h$
$9 = h$

Check.
Left side   Right side
21           $3 + 2h$
               $= 3 + 2(9)$
               $= 3 + 18$
               $= 21$ ✓

I used h to represent the number of hours I'll rent the skates and c to represent the cost.

I want to find out how many hours I can rent the skates with $21, so I substituted $c = 21$ into my equation.
What number plus 3 gives 21? $2h$ must be 18.
What number multiplied by 2 gives 18? h must be 9.

I checked to make sure that this solution works by substituting it into the equation.
My solution $h = 9$ is correct.
I can rent the skates for 9 h with $21.

Variables, Expressions, and Equations **285**

A Checking

3. Use inspection to solve the equations. Justify your reasoning, and check your solutions.

    a) $n + 6 = 13$

    b) $w - 11 = 22$

    c) $9p = 63$

    d) $2n + 3 = 15$

    e) $7r - 1 = 20$

4. Rana wrote the expression $4n + 3$ to describe the pattern below, where n represents the figure number.
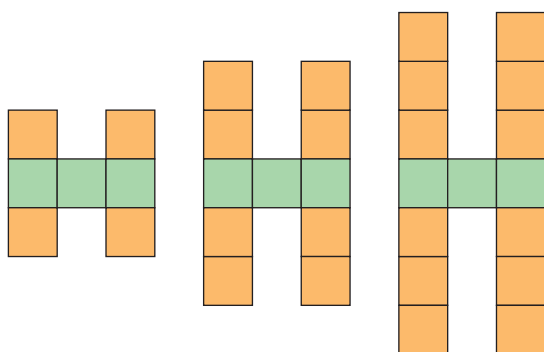
Figure 1   Figure 2   Figure 3

    a) Write an equation that describes the number of squares (*s*) needed to build each figure in this pattern.

    b) Rana built the figure in this pattern that has 23 squares. Write the equation that you must solve to determine the figure number.

    c) Solve your equation.

    d) Check your solution by using the equation.

    e) Check by drawing the figure and counting the squares.

B Practising

5. Solve each equation by inspection.

    a) $7b = 84$ **d)** $11 = q - 4$

    b) $8 + z = 30$ **e)** $2w + 1 = 17$

    c) $22 = m + 2$ **f)** $9n - 4 = 32$

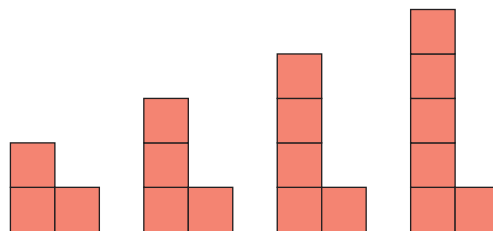**6. a)** Copy the following solution. Explain each step.

$$6 + 5m = 16$$
$$5m = 10$$
$$m = 2$$

    b) Explain how you can check to make sure the solution $m = 2$ is correct.

7. Can you build a figure that belongs in Rana's pattern in question 4 using 39 squares? Explain.

8. a) Write an equation that describes the number of squares needed to build each figure in this pattern.

    b) Suppose that you want to build a figure in this pattern using 24 squares. Write the equation you must solve to determine the figure number.

    c) Solve your equation.

    d) Check your solution by using the equation, and by drawing the figure and counting the squares.

    e) Use your equation to determine which figure number you could build using 21 squares. Describe this figure.

9. a) Suppose that you want to build the figure in this pattern that uses 28 pattern blocks. Write an equation you can solve to determine the figure number.
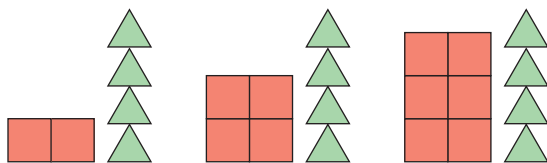
b) Solve your equation, and check your solution.

10. a) Suppose that you want to build the figure in this pattern that uses 30 squares. Write an equation you can solve to determine the figure number.
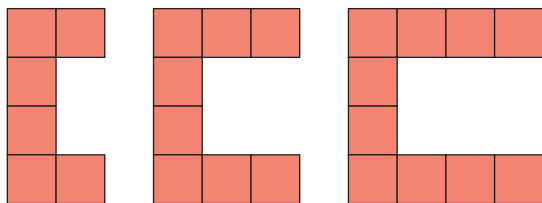
b) Solve your equation, and check your solution.

11. To rent a movie, a company charges $2 plus $1 per day.

a) Write an equation that describes the cost (c) of renting a movie in terms of the number of days (d).

b) Write an equation that represents the following question: "How many days can you rent a movie for $9?"

c) Solve your equation.

d) Check that your solution solves the problem.

12. Susan has $25. She is going to spend $4 on a book, then $3 per day on lunch.

a) Write an equation that represents the following question: "How many days can Susan buy lunch with this plan?"

b) Solve your equation, and check your solution.

13. Kevin and Zach are playing a number guessing game. Kevin says, "I am thinking of a number. If you double it and then subtract 1, the result is 7."

a) Write an equation that Zach could solve to find Kevin's number.

b) Explain the steps Zach could use to solve the equation.

C Extending

14. The sum of three consecutive whole numbers is 33. Create an algebraic equation, and solve to determine the numbers.

15. What is the greatest number of squares you can build with 100 toothpicks using this pattern?

Figure 1   Figure 2   Figure 3

16. What is the greatest number of triangles you can build with 100 toothpicks using this pattern?

Figure 1   Figure 2   Figure 3