7.4 Rotations

You will need
- centimetre grid paper
- a compass
- a protractor
- a ruler

GOAL
Identify the properties of a 2-D shape that stay the same after a rotation.

Learn about the Math

Kwami sees this starburst quilt in an art gallery and notices the rotated pattern.

? How can Kwami create his own starburst pattern?

Example 1: Constructing rotation images

Use geometry tools to construct rotations.

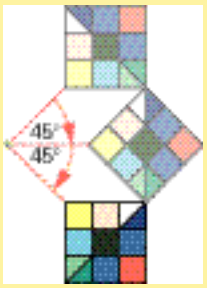
Kwami's Solution

I saw that the quilt's starburst is made of 8 rotated copies of a square design. I made 8 copies of it on separate pieces of grid paper.

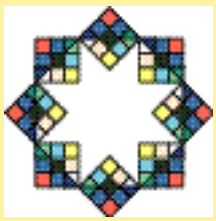
A circle has 360°, so I figured the 8 copies had to be 45° apart. On a large piece of paper, I used a protractor to draw a 45° angle.

I lined up the diagonal of a square along one of the angle arms. Then I moved the squares back and forth along the lines until they met at a corner. I glued the squares in place.

I set my compass to the distance that the corner of the square had to be from the turn centre.



I marked another 45° angle at the centre from the previous angle and used my compass to mark where the corner of the next square had to go. Then I glued the square in place.



I kept drawing new 45° angles, marking the corners, and gluing the squares until I finished the starburst.

Reflecting

1. Why was Kwami correct when he said that the copies of the square design had to be 45° apart? Where is the **centre of rotation**?
2. What properties of the squares changed after each rotation? What properties did not change?
3. Suppose that Kwami wants to make a quilt in which the square pattern has 10 rotated copies. What should he do differently?

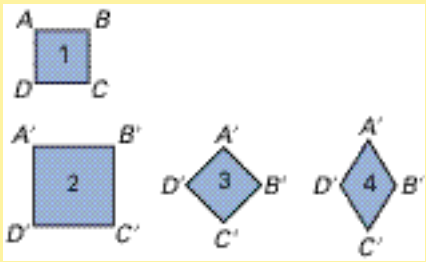
centre of rotation
a fixed point around which other points in a shape rotate in a clockwise (cw) or counterclockwise (ccw) direction; the centre of rotation may be inside or outside the shape

Work with the Math

Example 2: Recognizing rotation images

Which figures could be rotated images of figure 1?

Jody's Solution



Rotations do not change the size of a figure, so figure 2 cannot be a rotated image of figure 1.

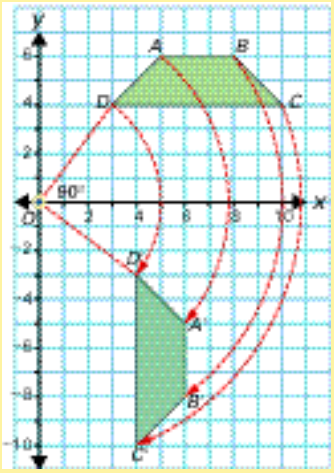
Figure 3 could be a rotated image of figure 1, since it is congruent to figure 1.

Rotations do not change the shape of a figure, so figure 4 cannot be a rotated image of figure 1.

Example 3: Rotating a shape on a coordinate grid

Plot points $A(5, 6)$, $B(8, 6)$, $C(10, 4)$, and $D(3, 4)$ on a Cartesian grid. Join the points to form a quadrilateral. Rotate the quadrilateral 90° cw about the origin, O.

Indira's Solution



I plotted the points and joined them to form a trapezoid. I drew a line from O to D. Using a protractor, I measured a 90° angle from line segment OD and drew another line segment, perpendicular to OD. I placed the point of a compass on O and the pencil on D. I drew an arc from D to meet the perpendicular line. I marked this point D' (the image of D after rotation).

I used the same method to find the other image points.

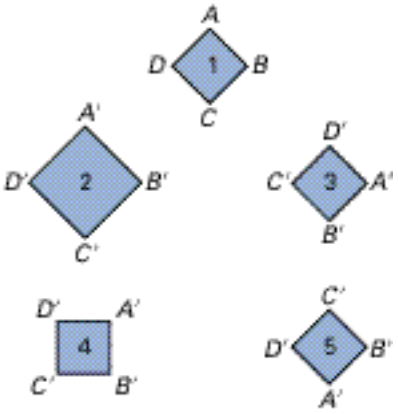
I joined the image points to form trapezoid $A'B'C'D'$.

The coordinates of the image are $A'(6, -5)$, $B'(6, -8)$, $C'(4, -10)$, and $D'(4, -3)$.

A Checking

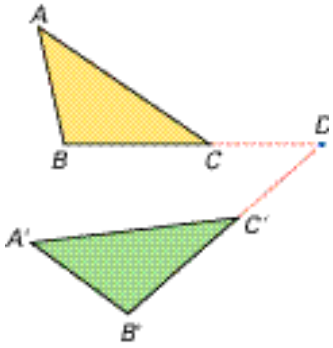
4. a) Which figures below could be rotated images of figure 1? Explain.

b) Look at the figures that cannot be rotated images of figure 1. Could they be images of figure 1 after a different transformation? Explain.



5. Your teacher tells you that the hour hand on an analog clock will rotate 45° while you write a math quiz. If the quiz starts at 9:00, what time does it end? What is the angle between the starting and ending positions of the minute hand?

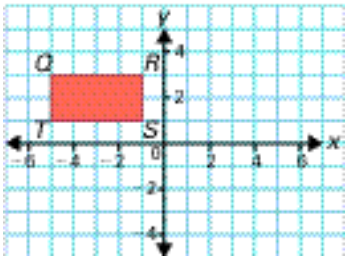
6. $\triangle A'B'C'$ is the image of $\triangle ABC$ after a rotation about the centre of rotation D. Suppose that you use a compass to draw a circle with centre D, and B is on the circle. Which other point must be on the circle? Explain.



7. The vertices of $\triangle ABC$ have coordinates $A(2, 3)$, $B(1, 5)$, and $C(-1, 1)$. Determine the coordinates of the image of $\triangle ABC$ after a 90° ccw rotation about the origin.

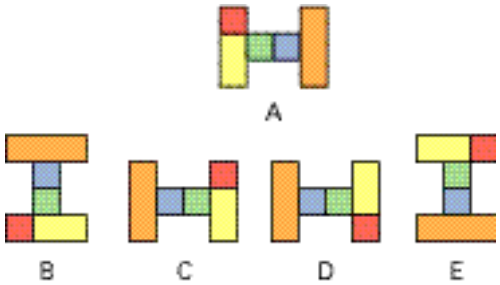
B Practising

8. Rotate rectangle $QRST$ 90° cw about the origin, O. Label the coordinates of the vertices of the image.



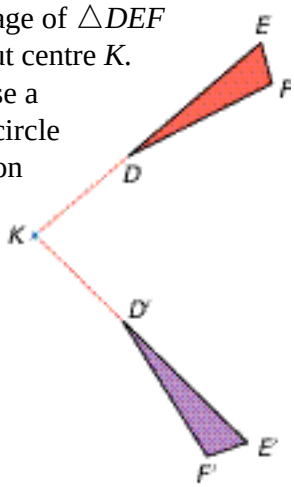
9. a) Which figure below is not the result of a rotation of figure A? Explain.

b) What transformation created the image that is not a rotation of figure A?



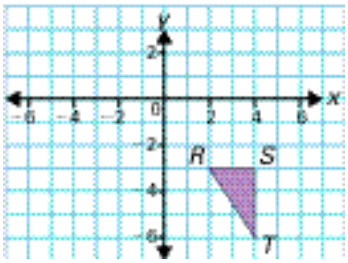
10. One of the rotated images in question 9 can also be formed by reflecting figure A. How can you form the image by reflection?

11. $\triangle D'E'F'$ is the image of $\triangle DEF$ after a rotation about centre K. Suppose that you use a compass to draw a circle with centre K. E is on the circle. Which other point must be on the circle? Explain.



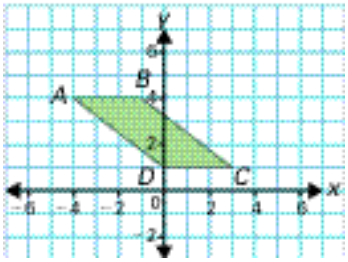
Use centimetre grid paper for questions 12 to 15.

12. a) Rotate $\triangle RST$ 270° cw about the origin, O. Label the coordinates of the vertices of the image.



b) Predict a different rotation that would move $\triangle RST$ to the same image as in part (a). Justify your prediction.

13. a) Rotate quadrilateral $ABCD$ 180° ccw about vertex D. Label the coordinates of the vertices of the image.



b) Predict a different rotation that would move quadrilateral $ABCD$ to the same image as in part (a). Justify your prediction.

14. a) Draw any triangle, and label its vertices X, Y, and Z.

b) Rotate $\triangle XYZ$ 360° cw about point X. Label the points of the image X', Y', and Z'.

c) What do you notice? Explain why this happens.

15. $\triangle E'F'G'$ with $E'(1, -1)$, $F'(1, 3)$, and $G'(4, 0)$ is the rotation image of $\triangle EFG$ after a 90° cw rotation about F. Determine the vertices of $\triangle EFG$.

16. a) Draw rhombus $PQRS$ with a 30° angle at vertex R and sides 2 cm long. How many degrees must you rotate $PQRS$ about vertex R so that the image touches the pre-image along an edge?

b) Rotate the rhombus about vertex R so that all the images touch along an edge, without overlapping. How many figures are in the design? What are the angles of rotation? What pattern do you see?

c) If the rhombus were drawn with an angle of 60° at vertex R, how many figures would be in the starburst? What would the angles of rotation be? Show how you know.

17. a) Plot $A(4, 5)$, $B(7, 5)$, $C(10, 2)$, $D(2, 2)$, and $R(5, 4)$. Join points A, B, C, and D to form a quadrilateral.

b) Rotate quadrilateral $ABCD$ 180° about point R. What are the coordinates of the vertices of the rotated image, $A'B'C'D'$?

C Extending

18. Use $A(0, 0)$, $B(0, 3)$, and $C(1, 3)$.

a) Draw and rotate $\triangle ABC$ 90° cw about point A to produce $\triangle A'B'C'$.

b) Rotate $\triangle A'B'C'$ 90° cw about point A to produce $\triangle A''B''C''$.

c) Is there a single rotation that will move $\triangle ABC$ directly to $\triangle A''B''C''$? If so, what is the angle of rotation? What is the centre of rotation?

19. Use $A(-5, 2)$, $B(-2, 3)$, and $C(-2, 1)$.

a) Reflect $\triangle ABC$ in the y-axis to produce $\triangle A'B'C'$. Then reflect $\triangle A'B'C'$ in the x-axis to produce $\triangle A''B''C''$.

b) Determine the coordinates of $\triangle A''B''C''$.

c) Is there a single transformation that will move $\triangle ABC$ to $\triangle A''B''C''$? Explain.