236 Chapter 7

Reflections

GOAL

Explore the properties of reflections of 2-D shapes.

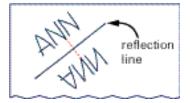
Learn about the Math

7.3

Andrea is decorating the door of her room.

How can you draw the reflection of your name without a mirror?

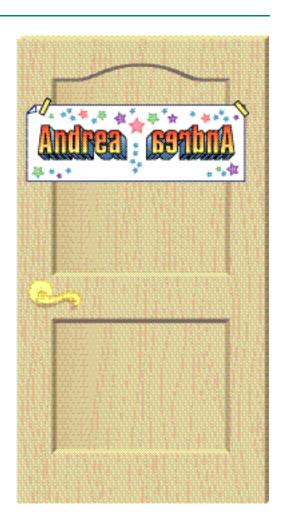
- **A.** On a piece of paper, print your name in capital letters on a slant.
- **B.** Draw a reflection line that is not horizontal or vertical.



- **C.** Use a transparent mirror to trace the image of your name.
- **D.** Connect three different points on your name with their image points.
- E. Use a ruler and a protractor to draw a more accurate reflection image of your name.

Reflecting

- **1.** How does the size and shape of a reflected image compare with the size and shape of the pre-image?
- **2.** What do you notice about the distances on each side of the reflection line?
- **3.** What do you notice about the angle between the reflection line and the line segments you drew?
- **4.** Why does using a ruler and protractor allow you to draw a more accurate reflection image?



You will need

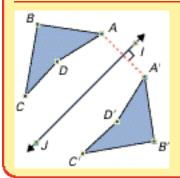
- centimetre grid paper
- a ruler
- a transparent mirror
- a protractor

Work with the Math

Example 1: Constructing a reflected image

Reflect quadrilateral ABCD in reflection line IJ.

James's Solution



I drew a line segment from point A, perpendicular to IJ, and continued my line segment beyond IJ. I measured the distance from A to IJ along my line segment. I measured the same distance on the other side of IJ, and labelled the point A'.

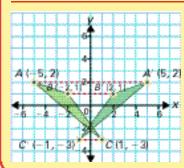
I used this method to find the other three image vertices.

I joined A', B', C', and D' to form the image. I noticed that the orientation of the image is opposite to the orientation of the pre-image.

Example 2: Drawing a reflected image using coordinates

Reflect $\triangle ABC$ in the *y*-axis.

Simon's Solution



A is 5 units to the left of the *y*-axis, so the image point A' is 5 units to the right of the *y*-axis at (5, 2).

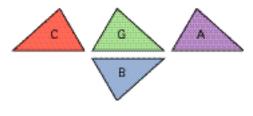
B is 2 units to the left of the *y*-axis, so the image point B' is 2 units to the right of the *y*-axis at (2, 1).

C is 1 unit to the right of the *y*-axis, so the image point *C'* is 1 unit to the left of the *y*-axis at (-1, -3).

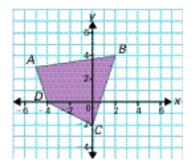
I joined the image points to form the image $\triangle A'B'C'$.

A Checking

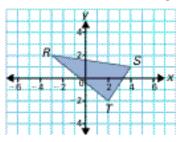
5. Which of figures A, B, and C is not a reflection of figure G? Explain.



6. a) Reflect quadrilateral *ABCD* in the *y*-axis. Determine the coordinates of the image quadrilateral.

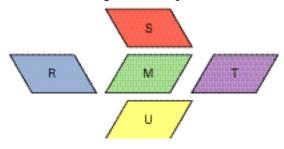


6. b) Reflect $\triangle RST$ in the *x*-axis. Determine the coordinates of the image triangle.

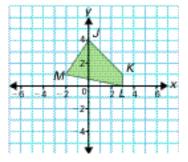


B Practising

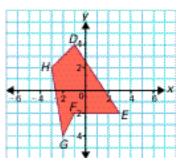
7. Which of figures R, S, T, and U are not reflections of figure M? Explain.



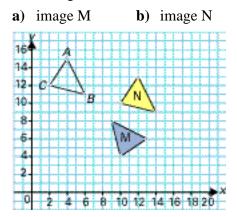
8. a) Reflect quadrilateral *JKLM* in the *x*-axis. Determine the coordinates of the image.



b) Reflect pentagon *DEFGH* in the *y*-axis. Determine the coordinates of the image.



9. Triangles M and N are images of $\triangle ABC$. Describe the transformation(s) that created each image.



10. The word MOM is a **palindrome**. This means that it is the same word when it is read backward. As well, MOM is the same word when it is reflected in a vertical line.

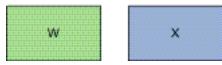


- a) Write the longest three words you can think of that are the same when they are reflected in a vertical line.
- **b)** Must these words also be palindromes? Explain.
- **11.** The word BOB is also a palindrome. As well, it is the same word when it is reflected in a horizontal line.

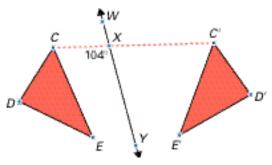


- a) Write the longest three words you can think of that are the same when they are reflected in a horizontal line.
- **b)** Must these words also be palindromes? Explain.

- **12**. The vertices of $\triangle ABC$ have coordinates A(-3, 0), B(1, 3), and C(2, -1). Determine the coordinates of the image of $\triangle ABC$.
 - **a**) after a reflection in the *y*-axis
 - **b**) after a reflection in the *x*-axis
- **13.** Figure X is the image of figure W.
 - a) Can you tell whether figure W was translated or reflected? Explain.



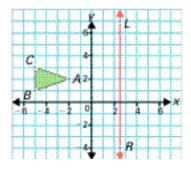
- **b**) Sketch the shapes in part (a). Label the vertices so that figure X is a reflection of figure W.
- **14. a)** Use centimetre grid paper to draw a trapezoid. Label the trapezoid *WXYZ*.
 - b) Draw a reflection line so that W' (the image point of vertex W) is 6 cm from W. Draw the reflected image of WXYZ, and label it W'X'Y'Z'.
 - c) What is the distance from *W* to the reflection line?
- 15. The reflected image of △*CDE* is △*C'D'E'*. Is line segment *WY* the reflection line? Explain, giving at least two reasons for your answer.



16. Determine the greatest four-digit number that is a lesser number when you reflect it in a vertical line.

G Extending

17. a) Reflect $\triangle ABC$ in the y-axis to produce the image $\triangle A'B'C'$.



- **b)** Reflect $\triangle A'B'C'$ in the line *LR* to produce the image $\triangle A''B''C''$.
- c) Determine the coordinates of $\triangle A''B''C''$.
- d) Is there a single transformation that moves △ABC to the image △A"B"C"? If so, describe the transformation. If not, explain why not.
- e) Will the result in part (d) be true for reflections in every pair of parallel lines? Write a hypothesis, and explore it using a number of examples.
- **18.** The vertices of $\triangle ABC$ have the coordinates A(-2, 0), B(0, 0), and C(0, 3). Draw a line through the points S(-3, 3) and T(3, 3).
 - a) Reflect $\triangle ABC$ in the *y*-axis. Then reflect the resulting image in the line *ST*. Determine the coordinates of the final image triangle.
 - **b)** Reflect $\triangle ABC$ in the line *ST*. Then reflect the resulting image in the *y*-axis. Determine the coordinates of the final image triangle.
 - c) Compare your results in parts (a) and (b). If you apply two reflections, one after the other, does the order in which you apply them matter? Write a hypothesis, and explore it using several examples.