7.2 Translations

You will need
- centimetre grid paper
- a ruler

GOAL
Recognize the image of a 2-D shape after a translation.

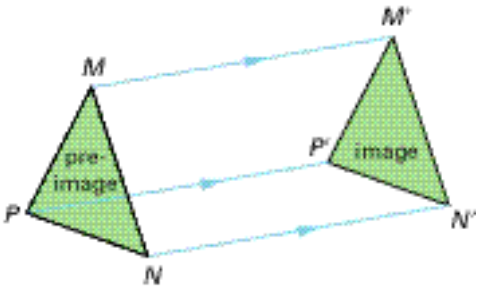
Learn about the Math

Jody is tiling a floor using square black and white tiles. She wants the floor to have a chessboard pattern. All the tiles are in a pile at $(-2, 0)$. Jody takes one tile at a time from the pile and slides it into position.

? How can Jody slide the tiles into position?

Communication Tip

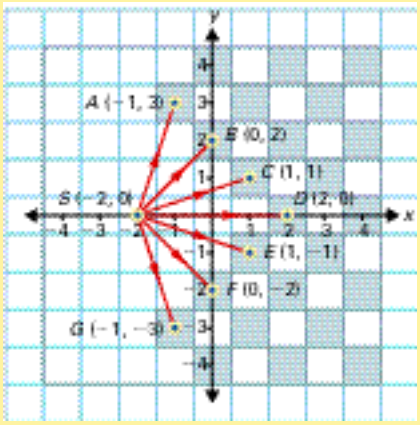
The new shape that is created when a shape is transformed is called the *image*. The original shape is called the *pre-image*. The vertices of the image are often labelled using the same letters as the pre-image, but with primes. (For M', say "M prime.") This shows which vertices of the image match which vertices of the pre-image. When M' is transformed, its image is M''.



Example 1: Describing translation images using a coordinate grid

Describe how Jody slides the tiles into position.

Jody's Solution



I began by moving some of the black tiles into position from their starting point at $S(-2, 0)$. I moved them along the arrows shown in the diagram. Here's how I moved each tile and the tile's final position:

A: 1 unit to the right and 3 units up to $A(-1, 3)$
B: 2 units to the right and 2 units up to $B(0, 2)$
C: 3 units to the right and 1 unit up to $C(1, 1)$
D: 4 units to the right to $D(2, 0)$
E: 3 units to the right and 1 unit down to $E(1, -1)$
F: 2 units to the right and 2 units down to $F(0, -2)$
G: 1 unit to the right and 3 units down to $G(-1, -3)$

Then I moved the rest of the tiles into position until the floor was complete.

Example 2: Determining translations using the coordinates of the starting point and endpoint

Determine how Jody slid each tile into position, using just the coordinates of the starting point and endpoint of the slide.

Indira's Solution

starting point: $S(-2, 0)$ endpoint: $A(-1, 3)$	The endpoint, A, has coordinates $(-1, 3)$. The starting point, S, has coordinates $(-2, 0)$.
$(-1) - (-2) = 1$ or $(-1) - (-2) = +1$	The difference between the x-coordinates is $+1$ or 1.
$3 - 0 = 3$ or $(+3) - 0 = +3$	The difference between the y-coordinates is $+3$ or 3.
	So, Jody slid the tile 1 unit to the right and 3 units up.

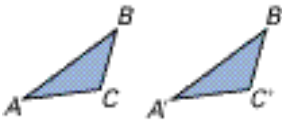
Reflecting

1. For the translation of a tile, draw an arrow from each vertex of the pre-image to the matching vertex of the image. What do the arrows have in common?

2. What properties of a pre-image and its image are the same after a translation? What properties are different? Include **orientation** in your answer.

orientation

the direction that a shape or an object is facing; for example, $\triangle ABC$ and $\triangle A'B'C'$ have the same orientation

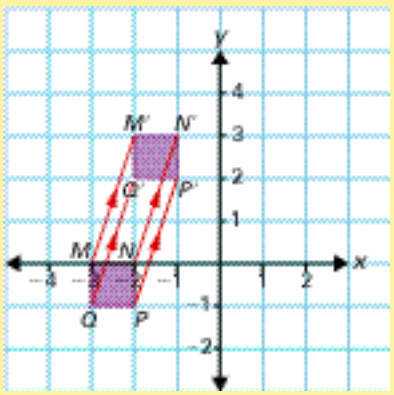


Work with the Math

Example 3: Describing translation images using a coordinate grid

Describe the effects of the translation shown below.

Kwami's Solution



From the diagram, the coordinates of M are $(-3, 0)$ and the coordinates of M' are $(-2, 3)$.

The difference between the x-coordinates is $(-2) - (-3) = +1$.

The difference between the y-coordinates is $(+3) - 0 = +3$.

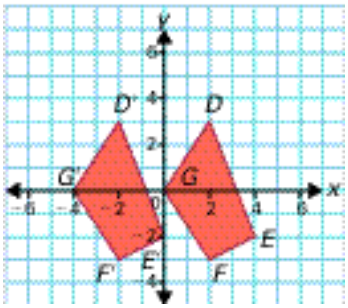
So, M moved 1 unit to the right and 3 units up to the image point M'.

Also, the other three vertices moved the same way, 1 unit to the right and 3 units up.

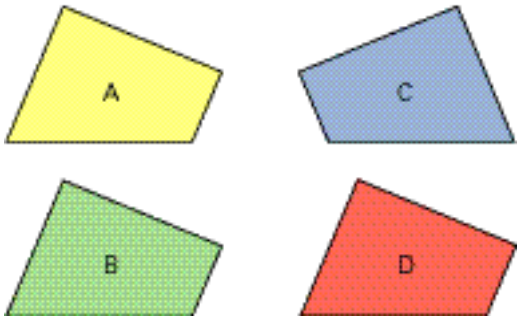
A Checking

3. Point A has coordinates (2, 4) on centimetre grid paper. It is translated 12 cm to the right and 3 cm up. What are the coordinates of the new location?

4. Describe the transformation that moved quadrilateral $DEFG$ to quadrilateral $D'E'F'G'$.



5. Which of figures B, C, and D is not a translation of figure A? Explain.

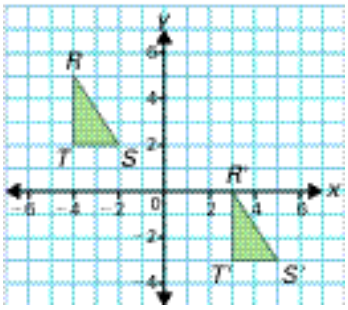


B Practising

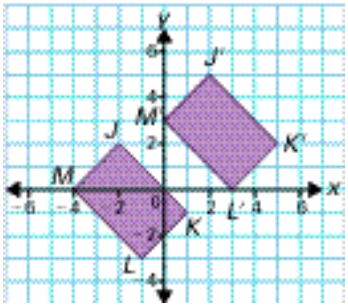
Use centimetre grid paper for the following questions.

6. The vertices of $\triangle ABC$ have coordinates $A(1, 2)$, $B(3, 5)$, and $C(3, -1)$. $\triangle ABC$ is translated 2 units to the right and 1 unit down. Determine the coordinates of the image triangle.

7. a) Describe the transformation that moved $\triangle RST$ to $\triangle R'S'T'$.

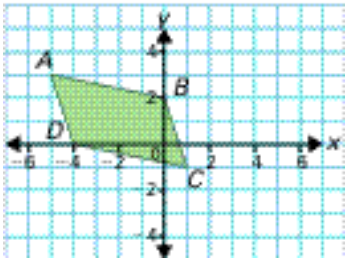


b) Describe the transformation that moved rectangle $JKLM$ to rectangle $J'K'L'M'$.

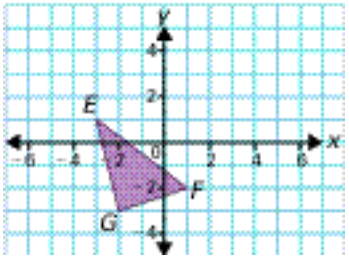


8. Copy each shape and translate it to determine the image coordinates.

a) Translate parallelogram $ABCD$ 3 cm down.



b) Translate $\triangle EFG$ 2 cm to the left and 1 cm up.



9. The vertices of square $ABCD$ are $A(-1, -1)$, $B(1, -1)$, $C(1, 1)$, and $D(-1, 1)$. Square $ABCD$ is translated 3 cm to the left and 2 cm up. Determine the coordinates of all the vertices of the image square.

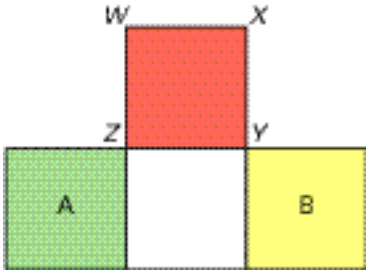
10. **a)** Draw any triangle, $\triangle DEF$, on a coordinate grid. Record the coordinates of each vertex.

b) Transform $\triangle DEF$ 5 cm to the left and 4 cm down. Label the image $\triangle D'E'F'$.

c) Determine the coordinates of D', E', and F'.

11. $\triangle DEF$ has a base, DE, that is 7 cm long. Describe how you would translate $\triangle DEF$ so that D' is the same point as E. Draw a diagram to illustrate your description.

12. Square $WXYZ$ has sides that are 2 cm long. Squares A and B are translation images of square $WXYZ$. Describe how square $WXYZ$ was translated to create squares A and B.



13. The vertices of $\triangle ABC$ have coordinates $A(3, -2)$, $B(0, 0)$, and $C(2, 2)$.

a) $\triangle ABC$ is translated 3 cm to the right and 2 cm down. Determine the coordinates of the image triangle. Label the image triangle $\triangle A'B'C'$.

b) $\triangle A'B'C'$ is translated 1 cm to the right and 3 cm up. Determine the coordinates of the image triangle. Label the image triangle $\triangle A''B''C''$.

c) Describe a single translation that moves $\triangle ABC$ directly to $\triangle A''B''C''$.

14. Nathan has a paper route in a part of town where all the streets run either north-south or east-west. From his home, he travels 4 blocks north, 3 blocks east, 7 blocks south, 2 blocks east, 5 blocks north, 3 blocks west, and 2 blocks south. At the end of his route, how many blocks is he from his home, and in what direction?

15. The vertices of $\triangle ABC$ have coordinates $A(2, 0)$, $B(3, 0)$, and $C(2, 2)$.

a) Translate $\triangle ABC$ 2 cm to the right and 1 cm down. Then translate the resulting image 1 cm to the right and 2 cm up.

b) Start again with $\triangle ABC$. Translate $\triangle ABC$ 1 cm to the right and 2 cm up. Then translate the resulting image 2 cm to the right and 1 cm down.

c) Compare your results in parts (a) and (b). If you apply two translations, one after the other, does the order in which you apply them matter? Write a hypothesis, and explore it using several examples.

C Extending

16. $\triangle ABC$ is translated 3 units to the left and 2 units down. The vertices of the image triangle, $\triangle A'B'C'$, have coordinates $A'(1, -1)$, $B'(3, -2)$, and $C'(2, 1)$. Determine the coordinates of the vertices of $\triangle ABC$.

17. $\triangle ABC$ is translated 2 units to the right and 1 unit up to produce $\triangle A'B'C'$. Then $\triangle A'B'C'$ is translated 3 units to the right and 2 units down to produce $\triangle A''B''C''$. The coordinates of three of the vertices are $A(0, 0)$, $B'(6, 0)$, and $C''(6, 0)$. Determine the coordinates of all the other vertices of the three triangles.