

# 6.5

## Integer Addition Strategies

### You will need

- a calculator with a sign change key

### GOAL

Learn integer addition strategies.

### Learn about the Math

Miguel and his friends play soccer. The coach tracks the goals scored while each team member is playing.

- Each goal scored for the team counts as  $+1$ .
- Each goal scored against the team counts as  $-1$ .
- These goals are added through the season to produce a  $+/-$  score for each player.

Here are the current  $+/-$  scores for Miguel and his friends.

Player	Miguel	Paul	Lloyd	Sam	Matthew
$+/-$ score	$+18$	$-23$	$-31$	$+21$	$+13$



### ? What is the overall $+/-$ score for the group of soccer players?

You can **regroup** the integers to have the positive and the negative integers together.

### regroup

change the order of terms in an arithmetic expression to form groups

### Example 1: Separating positive and negative sums

Add  $(+18) + (-23) + (-31) + (+21) + (+13)$ .

### Miguel's Solution

$$\begin{aligned}
 & (+18) + (-23) + (-31) + (+21) + (+13) \\
 = & \boxed{(+18) + (+21) + (+13)} + \boxed{(-23) + (-31)} \\
 = & (+52) + (-54) \\
 = & \boxed{(+52) + (-52)} + (-2) \\
 = & 0 + (-2) \\
 = & (-2)
 \end{aligned}$$

I can group all the positives together and all the negatives together.

The sum of the positives is  $(+52)$ . The sum of the negatives is  $(-54)$ .

$$(-54) = (-52) + (-2)$$

$$(+52) + (-52) = 0$$

The answer is  $(-2)$ .



### Example 2: Regrouping integers to produce more manageable sums

Add  $(+18) + (-23) + (-31) + (+21) + (+13)$ .

#### Paul's Solution

$$\begin{aligned} & (+18) + (-23) + (-31) + (+21) + (+13) \\ = & \boxed{(+18) + (+13)} + (-31) + \boxed{(-23) + (+21)} \\ = & \boxed{(+31)} + (-31) + (-2) \\ = & 0 + (-2) \\ = & (-2) \end{aligned}$$

I noticed that  $(-31)$  balances  $(+18) + (+13) = (+31)$ . I also noticed that  $(+21)$  and  $(-23)$  can be added to get  $(-2)$ .  $(+31)$  and  $(-31)$  are opposites, so they add to 0.

The answer is  $(-2)$ .

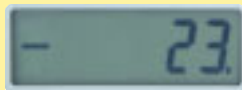


### Example 3: Using a calculator with a sign change key

Calculate  $(+18) + (-23) + (-31) + (+21) + (+13)$ .

#### Romona's Solution

18  $\boxed{+}$  23  $\boxed{+/-}$



$\boxed{+}$  31  $\boxed{+/-}$   $\boxed{+}$  21  $\boxed{+}$  13  $\boxed{=}$



I entered 18. Then I pressed  $\boxed{+}$  to show that the next number will be added to it.

Next I entered 23 and pressed the  $\boxed{+/-}$  key to change the sign. This changed  $+23$  to  $-23$ .

I used  $\boxed{+/-}$  and  $\boxed{+}$  to enter the rest of the numbers. I pressed  $\boxed{=}$  to complete the calculation.

The answer is  $(-2)$ .



## Reflecting

1. Miguel used separate positive and negative sums to get the final overall score. Explain why his strategy will work with any integer sum.
2. Describe how mental math skills helped Paul work with more manageable sums compared to Miguel's strategy. Will Paul's strategy work for all integer sums?
3. Compare the three strategies used by Miguel, Paul, and Romona. How are they alike? How are they different?

## Work with the Math

### Example 4: Adding positive and negative integers

Add  $(-114) + (+35) + (+11) + (-15) + (-20) + (+14)$ .

#### Solution A: Regrouping

$$\begin{aligned}
 & (-114) + (+35) + (+11) + (-15) + (-20) + (+14) \\
 = & (-114) + (+11) + (+14) + (+35) + (-15) + (-20) \\
 = & (-114) + (+11) + (+14) + 0 \\
 = & (-114) + (+11) + (+14) \\
 = & (+11) + (-114) + (+14) \\
 = & (+11) + (-100) \\
 = & (-89)
 \end{aligned}$$

#### Solution B: Positive and negative sums

$$\begin{aligned}
 & (-114) + (+35) + (+11) + (-15) + (-20) + (+14) \\
 = & (-114) + (+35) + (-35) + (+11) + (+14) \\
 = & (-149) + (+60) \\
 = & (-89)
 \end{aligned}$$

The answer is  $(-89)$ .

### Example 5: Using a calculator to add positive and negative integers

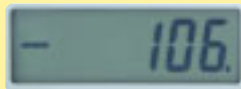
Calculate  $(+379) + (-106) + (-241)$ .

#### Solution

1. Enter 379 and then press  $\boxed{+}$ .



2. Enter 106 and then press  $\boxed{+/-}$ .



3. Press  $\boxed{+}$ . Enter 241. Press  $\boxed{+/-}$ .



4. Press  $\boxed{=}$ . The answer is  $(+32)$ .



## A Checking

4. Calculate, using the method indicated.
- $(-40) + (+55) + (+5) + (-40) + (-10)$   
Use regrouping to make more manageable sums.
  - $(-13) + (+8) + (-12) + (+10) + (+9)$   
Use separate positive and negative sums.
  - $(+225) + (-311) + (+110) + (-97)$   
Use a calculator.

## B Practising

5. Verify that these expressions all have the same sum.
- $(-5) + (-2) + (-3) + (+5)$
  - $(-5) + (+5) + (-2) + (-3)$
  - $(-2) + (-3) + (+5) + (-5)$
  - $(+5) + (-2) + (-3) + (-5)$

6. Consider the following expression:

$$(+4) + (-3) + (+1) + (+6) + (-2)$$

- Add the integers in the order in which they appear.
- Group together all the positive integers, and add them. Group together all the negative integers, and add them. Now add these two sums.
- Why do you get the same answer when you regroup and when you add the terms in the order in which they appear?

7. Calculate each sum.

- $(-12) + (+2) + (-5)$
- $(+23) + (-14) + (-7)$
- $(-18) + (+5) + (+18)$
- $(+7) + (-3) + (-13) + (+6)$
- $(-21) + (-30) + (+50) + (+10)$
- $(-10) + (+48) + (-38) + (-9)$

8. Fill in each  $\square$  with a different two-digit integer to make the equation true. Find two different solutions.

$$\square + \square + \square + \square + \square = (+4)$$

9. Add. Use the strategy of your choice.

- $(+8) + (-4) + (+3) + (-5) + (-6)$   
 $+ (+4) + (+1) + (+5)$
- $(-10) + (-15) + (+15) + (+20)$
- $(+45) + (-35) + (+15) + (-25)$   
 $+ (+20) + (-5)$
- $(-278) + (+415)$
- $(+426) + (-242) + (-318)$

10. A hockey player had a  $+/-$  score of  $+11$ . The following table shows how it changed over seven games. What is the player's  $+/-$  score now?

Game	1	2	3	4	5	6	7
$+/-$ score	-1	+4	-3	-2	+5	0	+1

11. Samantha bought some stock for \$21 a share. She decided to sell the stock if it rose to \$35 a share or dropped to \$11 a share. Based on the price changes below, did Samantha sell the stock? Explain.

Week	1	2	3	4	5	6
Price change (\$)	+10	+3	-12	-6	-15	+8

12. At 10:00, the position of a submarine is shown. Its changes in depth were recorded every hour in the table. What is the depth of the submarine at 15:00?

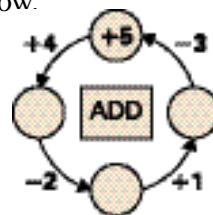


Time	11:00	12:00	13:00	14:00	15:00
Change in depth (m)	-53	-31	+18	-64	+85

## **C** Extending

13. a) Copy the diagram below.

Start with  $+5$  in the top circle. Fill in the other circles by following the arrows and adding the indicated integers.



- Why is the final sum  $+5$  after you finish the last addition?
- Copy the diagram again. Replace the numbers on the arrows with four different two-digit integers, so that you still end up with  $+5$ .