6.3 Adding Integers Using the Zero Principle

You will need
- red and blue counters

GOAL
Use the zero principle, with and without models, to add integers.

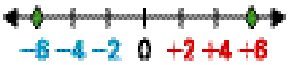
Learn about the Math

In a coin tossing experiment, Paul gained 1 point (+1) when he tossed Heads. He lost 1 point (−1) when he tossed Tails.

The following table shows Paul's results of 11 tosses.

Toss number	1	2	3	4	5	6	7	8	9	10	11
Result (+1) or (−1)	−1	−1	+1	+1	+1	−1	+1	−1	+1	−1	−1

opposite integers
two integers the same distance away from zero; for example, +6 and −6 are opposite integers



? How can you add integers to calculate Paul's score after 11 tosses?

You can use a blue counter to represent (−1) and a red counter to represent (+1). The integers (+1) and (−1) are **opposite integers**.

Adding (−1) and (+1) gives a net result of zero.

This is the **zero principle**.

zero principle
two opposite integers, when added, give a sum of zero; for example, (−1) + (+1) = 0

Example 1: Modelling the sum with counters

Use counters to calculate Paul's score after 11 tosses.

Paul's Solution

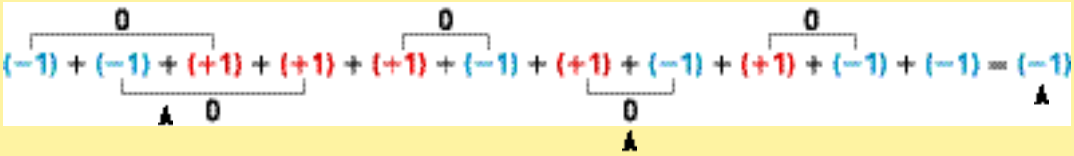
I modelled my first 11 tosses using counters.
I used blue counters to represent (−1) and red counters to represent (+1).

I changed the order to get pairs of blue and red counters.
In each pair, (+1) paired with (−1) is 0.
One blue counter was left over.
The answer is (−1).

Example 2: Using +1s and −1s

Use positive 1s and negative 1s to calculate Paul's score after 11 tosses.

Fawn's Solution



I found pairs of positive 1s and negative 1s.

I added the +1 and the −1 in each pair to get 0.

There was a single −1 left over.

Example 3: Combining integers greater than +1 and less than −1

Use integers greater than +1 and less than −1 to calculate Paul's score after 11 tosses.

Miguel's Solution

$$\underbrace{(+1)+(+1)+(+1)+(+1)+(+1)}_{(+5)}+\underbrace{(-1)+(-1)+(-1)+(-1)+(-1)}_{(-5)}+(-1)$$

I wrote all the positive 1s first.

Then I wrote all the negative 1s.

$$\begin{aligned} &(+5)+(-5)+(-1)\\ &=0+(-1) && \text{The zero principle says that } (-5)+(+5)=0.\\ &=(-1) && (-1) \text{ was left over.} \end{aligned}$$

Reflecting

1. a) How are Paul's and Fawn's solutions alike?

b) How are they different?

2. Miguel used the idea that the sum of any two opposite integers is always zero. Verify Miguel's solution using counters.

Communication Tip

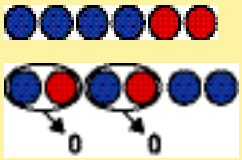
- Read the integer +1 as "positive 1" and the integer −1 as "negative 1."
- Integer expressions often have brackets around the integers.
- Do not confuse the sign of an integer with the operation of addition or subtraction. For example, to add +2 and −4, write (+2) + (−4). This means "positive 2 plus negative 4."

Work with the Math

Example 4: Using the zero principle

Add $(-4) + (+2)$.

Solution A: Using counters



The answer is (-2).

Solution B: Using numbers greater than +1 and less than −1

$(-4) + (+2) = (-2) + \boxed{(-2) + (+2)}$

$= (-2)$ (the boxed pair $\rightarrow 0$)

The answer is (-2).

A Checking

3. Add each expression using counters and numbers.

	Expression	Using counters	Using numbers
a)	$(-3) + (+2)$		
b)	$(-4) + (+6)$		
c)	$(+5) + (-6)$		
d)	$(-5) + (+7)$		
e)	$(+2) + (-8)$		
f)	$(-1) + (-9)$		

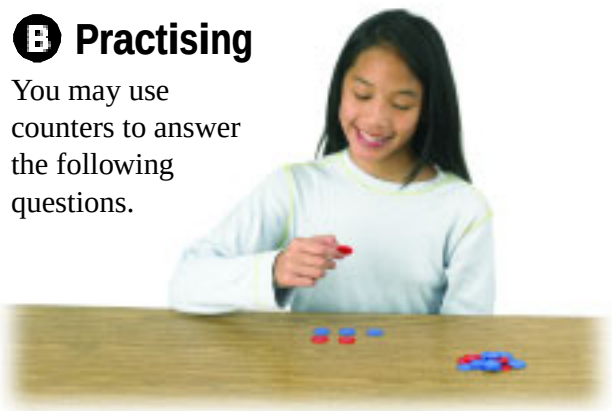
4. Use mental math to determine each sum.

a) $(+3) + (-3) =$ ▢

b) $(-7) + (+7) =$ ▢

B Practising

You may use counters to answer the following questions.



5. Complete.

a) $(-3) + (-2) =$ ▢

b) $(+2) + (-2) =$ ▢

c) $(-4) + (+1) =$ ▢

d) $(-7) + (+6) =$ ▢

e) $(-5) + (-2) =$ ▢

f) $(-5) + (+2) =$ ▢

6. Explain why $(-25) + (+25) = 0$.

7. The following patterns are based on adding integers. Continue each pattern. Then write a rule to describe each pattern.

a) $0, -1, -2, -3, -4,$ ▢, ▢, ▢

b) $-3, -2, -1, 0,$ ▢, ▢, ▢

8. Fill in each ▢ with $+1$ or -1 to make each statement true.

a) $(+1) +$ ▢ $+$ ▢ $= (-1)$

b) $(-1) +$ ▢ $+$ ▢ $= (+1)$

c) $(+1) +$ ▢ $+$ ▢ $+$ ▢ $+$ ▢ $= (-1)$

d) $(+1) +$ ▢ $+$ ▢ $+$ ▢ $+ (+1) = (-1)$

9. Complete.

a) $(-3) + (+3) + (+5) =$ ▢

b) $(-1) + (-2) + (-1) =$ ▢

c) $(+2) + (+1) +$ ▢ $= (-1)$

10. Use $=$, $<$, or $>$ to make each statement true.

a) $(-1) + (-2) \square (-4)$

b) $(+2) + (-5) \square (-3)$

c) $(-3) + (+6) \square (+2)$

d) $(+5) + (-7) \square (-2)$

e) $(-2) + (-4) \square (-5)$

f) $(-2) + (+1) \square 0$

11. Using $+1$ and -1 only, create an addition question that has each sum. Use at least four numbers for the question. Check your work using counters.

a) $+3$ **b)** -2 **c)** 0 **d)** -1

12. a) Calculate the sum. You can use counters or numbers.
$(+1) + (+1) + (-1) + (+1) + (-1) + (-1) + (+1) + (+1) + (-1)$

b) Which method did you choose? Why?

13. Fill in each $\square$ with an integer to make the equation true. Show three different solutions.

$\square + \square + \square = (-5)$

14. Explain why you cannot complete this equation using only $+1$s or -1s.

$(+1) + \square + \square + \square = (+1)$

15. In a Magic Square, all rows, columns, and diagonals have the same sum. No number appears more than once.

a) This Magic Square uses integers from -6 to $+2$. Verify that the rows, columns, and diagonals all have the same sum. The sum of the third column is shown.

+1	−6	−1
−4	−2	0
−3	+2	−5

(−1) + 0 + (−5) = (−6)

b) This Magic Square uses integers from -1 to $+7$. Complete it. Check that all the sums are the same.

+2		
+7	+3	−1

c) This Magic Square uses integers from -4 to $+4$. Complete it. Check that all the sums are the same.

+3	−4	
−2		
	+4	

d) Create a Magic Square that uses integers from -10 to -2.

C Extending

16. State whether each statement is true or false. Explain your reasoning.

a) The sum of two positive integers is positive.

b) The sum of two negative integers is negative.

c) The sum of a negative integer and a positive integer is always positive.

17. Continue each pattern. Write a rule to describe the pattern.

a) $0, +1, -1, +2, -2, +3, -3, \square, \square, \square$

b) $-1, 0, -1, -1, -2, -3, -5, -8, \square, \square, \square$

18. Without using a calculator, determine the sum of all the integers from -50 to $+50$. Describe your strategy.

19. a) Calculate the average daily high temperature for the four days.

Four-day weather forecast	High temperature (°C)	Low temperature (°C)
Wednesday	+10	0
Thursday	+5	−6
Friday	+9	−7
Saturday	+8	−3

b) Calculate the average daily low temperature for the four days.

c) What is the range between the highest high and the lowest low?