5.6 Calculating the Area of a Complex Shape

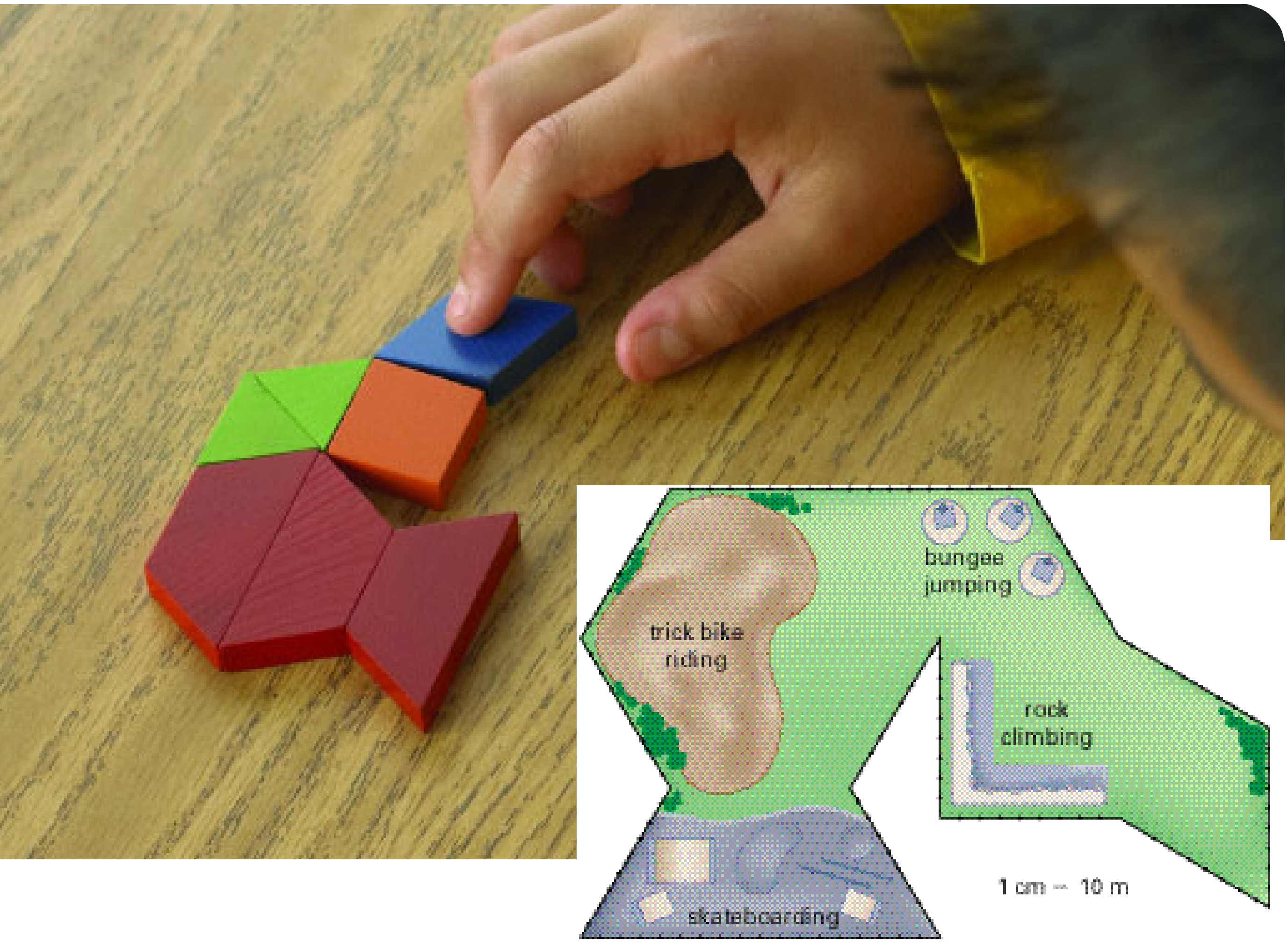
You will need
- pattern blocks
- a ruler
- a calculator

GOAL
Calculate the area of an irregular 2-D shape by dividing it into simpler shapes.

Learn about the Math

Chang is designing an extreme sports park for Adventurepark. He is using pattern blocks to try different designs.

? How can you calculate the area of the extreme sports park?



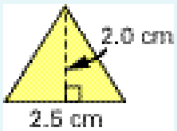
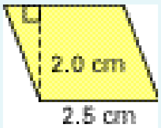
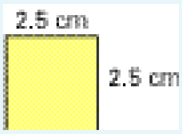
Example 1: Calculating the area by using simpler shapes

What is the area of the extreme sports park?

Chang's Solution

I used three trapezoids, two triangles, one parallelogram, and one square to make the complex shape. I know how to calculate the area of each of these simpler shapes.

I measured the base and the height of each pattern block, and labelled the measurements on sketches. Then I used a formula to calculate each area. I added the areas to find the total area of the complex shape.

Simpler polygon	Area of simpler polygon	Total area
	$A = (a + b) \times h \div 2$ $= (5.0 \text{ cm} + 2.5 \text{ cm}) \times 2.0 \text{ cm} \div 2$ $= 7.5 \text{ cm} \times 2.0 \text{ cm} \div 2$ $= 7.5 \text{ cm}^2$	There are three trapezoids with the same area. $3 \times 7.5 \text{ cm}^2 = 22.5 \text{ cm}^2$
	$A = b \times h \div 2$ $= 2.5 \text{ cm} \times 2.0 \text{ cm} \div 2$ $= 2.5 \text{ cm}^2$	There are two triangles with the same area. $2 \times 2.5 \text{ cm}^2 = 5.0 \text{ cm}^2$
	$A = b \times h$ $= 2.5 \text{ cm} \times 2.0 \text{ cm}$ $= 5.0 \text{ cm}^2$	5.0 cm^2
	$A = l \times w$ $= 2.5 \text{ cm} \times 2.5 \text{ cm}$ $= 6.25 \text{ cm}^2$	6.25 cm^2
To find the total area of the complex shape, add the smaller areas. $1 \text{ cm} = 10 \text{ m}$, so $1 \text{ cm}^2 = 10 \text{ m} \times 10 \text{ m}$, or 100 m^2		38.75 cm^2 The area of the sports park is about 3880 m^2.

Reflecting

1. Explain how starting with simpler polygons helps you calculate the area of a complex polygon. Use Chang's example to support your answer.

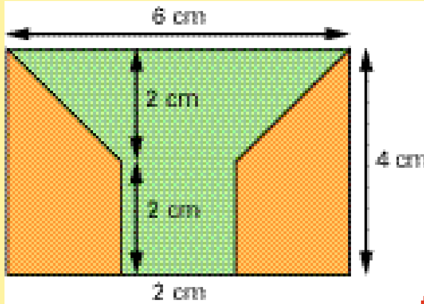
2. Chang calculated the area of one trapezoid and the area of one triangle. Explain why he did not need to calculate the areas of the other two trapezoids and the other triangle.

3. Divide the complex polygon into a different set of simpler shapes. What is the sum of their areas?

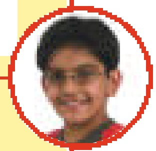
Work with the Math

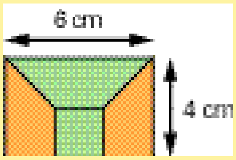
Example 2: Calculating the area of an irregular shape

Calculate the total area of the orange shapes in this rectangle.



Ravi's Solution





I divided the green shape into a trapezoid and a square.

Area of rectangle
$= l \times w$
$= 6\text{ cm} \times 4\text{ cm}$
$= 24\text{ cm}^2$

Area of trapezoid
$= (a + b) \times h \div 2$
$= (6\text{ cm} + 2\text{ cm}) \times 2\text{ cm} \div 2$
$= 8\text{ cm} \times 2\text{ cm} \div 2$
$= 8\text{ cm}^2$

Area of square
$= l \times w$
$= 2\text{ cm} \times 2\text{ cm}$
$= 4\text{ cm}^2$

I used formulas to calculate the areas of the shapes.
The area of the rectangle is 24 cm^2.
The area of the trapezoid is 8 cm^2.
The area of the square is 4 cm^2.

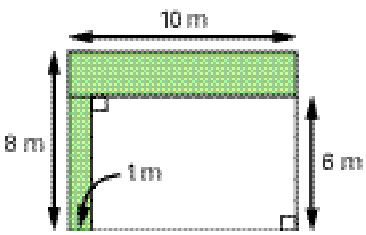
Area of rectangle minus area of trapezoid and square
$= 24\text{ cm}^2 - (8\text{ cm}^2 + 4\text{ cm}^2)$
$= 24\text{ cm}^2 - 12\text{ cm}^2$
$= 12\text{ cm}^2$

I subtracted the area of the trapezoid and the area of the square from the area of the rectangle.
The total area of the orange shapes is 12 cm^2.

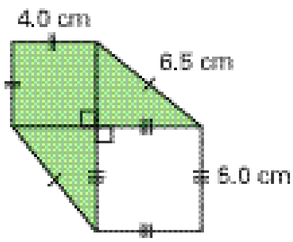
A Checking

4. Calculate the area of the green shapes in each diagram. Show your work.

a)



b)

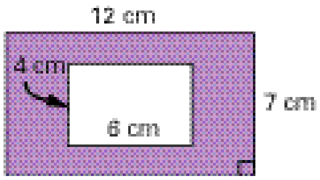


B Practising

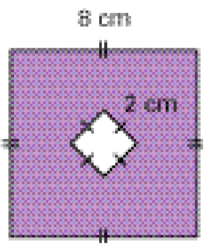
5. Use pattern blocks to create an irregular polygon. Trace your polygon on paper, and calculate its total area.

6. Calculate the area of the purple part of each diagram.

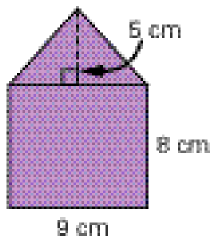
a)



b)



c)



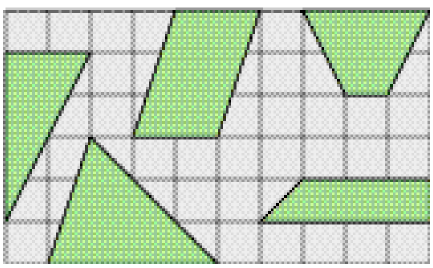
7. Heidi is designing a two-lane running track for an athletic club. The track will go around a rectangle that will have an area of 3600 m^2. The length and width of the rectangle must be whole numbers of metres.

a) Draw five possible tracks.

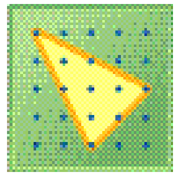
b) Which track is the longest?

c) Which track is the shortest?

8. The following diagram shows a picnic area. Each square represents 1 m^2. The green areas are grass. The grey areas need to be paved. If a paving company charges $12 to pave 1 m^2, how much will the company charge to pave all the grey areas?

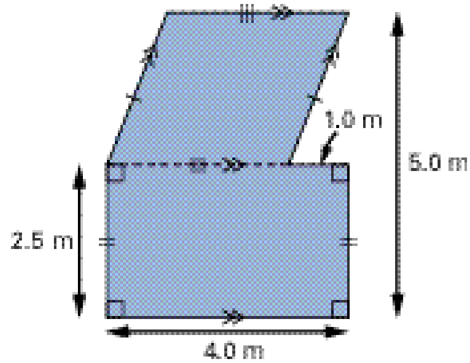


9. What is the area of the triangle on this geoboard?



10. Calculate the area of each polygon.

a)



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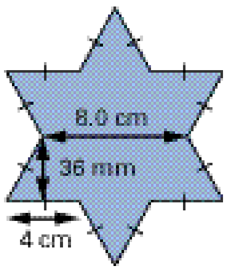

C Extending

11. Richard built a cement barrier that is 100 blocks long and 1 block high. He plans to paint the front, top, and back of each block, as well as any other visible sides. He does not plan to paint the sides that touch one another or the sides that are on the bottom. What is the total area of the surfaces that Richard will paint? (*Hint:* Start with four blocks, then consider ten blocks, and so on.)