4.3

Using a Table of Values to Represent a Sequence

You will need

- counters
- a calculator
- linking cubes

▶ GOAL

Use tables of values to represent number sequences.

Learn about the Math



Kaitlyn and Tynessa used counters to show a sequence of square numbers. They want to know whether they have enough counters for the 6th figure in the sequence.

? How can you determine the number of counters needed without building the figure?

Communication Tip

- The figure number is called the term number because it tells the position of the term in the sequence. The term numbers in Kaitlyn and Tynessa's sequence start at 1.
- The number of counters needed for each term number is called the term value.

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Example 1: Using a table of values to represent and analyze a sequence

Make a table of values. Determine the number of counters needed to model the 6th square number in the sequence.

Tynessa's Solution

Term number (figure number)	Picture	Term value (number of counters)	
1	88	1	-\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
2	88	4	$\sqrt{5} + 5$
3	888	9	+ 7
4	8000 8000 8000	16	+ 9
5	00000 00000 00000 00000	25	+ 1
6	000000 000000 000000 000000	36	

I noticed that when you go down the "Term value" column, you add the next odd number. I can write this pattern rule as "Start with 1 and add the next greatest odd number."

The value for the 6th square is 25 + 11 = 36.

This pattern rule uses the previous term in the sequence.

Term number	Rule	Term value
1	$1 \times 1 = 1^2$	1
2	$2 \times 2 = 2^2$	4
3	$3 \times 3 = 3^2$	9
4	$4 \times 4 = 4^2$	16
5	$5\times 5=5^2$	25
6	$6\times 6=6^2$	36

I also noticed that the term value is the square of the term number. I can write this rule:

term value = term number²

The value for the 6th square is 6² or 36.

This rule uses the position of the term in the sequence.

Reflecting

- **1.** How are Tynessa's tables of values alike? How are they different?
- **2**. The pattern rule in the first table uses an "adding on" strategy. What strategy was used for the rule in the second table?
- **3**. Which table would you use to calculate the 20th term in the sequence? Explain why.

Work with the Math

Example 2: Using a table of values to solve a problem

The rule to build a toothpick pattern is "Start with a square, and add 3 toothpicks each time to make another square." How many toothpicks do you need to build the 5th figure and the 20th figure in the pattern?

Colin's Solution

Term number (figure number)	Picture	Term value (number of toothpicks)
1	\Diamond	4
2	\Diamond	7
3	\otimes	10
4	$\langle \rangle \rangle$	13
5	$\langle \rangle \rangle$	16

Term number	Rule	Term value
1	× 3 + 1	4
2	× 3 + 1	7
3	× 3 + 1	10
4	× 3 + 1	13
5	× 3 + 1	16

My picture shows 16 toothpicks in the 5th figure. The rule for the number pattern is "Start with 4 and add 3 each time."

You can calculate the number of toothpicks for the 20th figure by starting with 4 and adding on 3 a total of 19 times.

$$4 + (3 \times 19) = 61$$

Another way to solve the problem is to figure out how the term number and the term value are related.

The value of each term is 1 more than three times the term number. This rule is:

term value = $(3 \times \text{term number}) + 1$ For the 20th term, $(3 \times 20) + 1 = 61$.

You get the same answer with both methods. There are 61 toothpicks in the 20th term.

A Checking

- **4. a)** Complete the table of values for the pattern shown.
 - **b)** Write a rule that tells how the value of each term can be calculated from the previous term in the sequence.
 - c) Write a rule that tells how the value of each term can be calculated from its term number.
 - **d)** Predict the value of the 8th term in the sequence.

Term number (figure number)	Picture	Term value (number of stars)
1	≱	3
2	3	5
3	教	7
4		
5		

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5. Peter and Heidi are looking at this table of values. Peter says that the pattern rule is "Start with 3 and add 3."

Term number	Term value
1	3
2	6
3	9
4	12
5	15

Heidi says that the pattern rule is "Multiply the term number by 3." Who is right? Explain.

B Practising

6. Copy and complete the table of values.

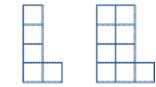
Term number (figure number)	Picture	Term value (number of tiles)
1		6
2		11
3		16
4		
5		

- 7. Make a table of values for each sequence. Include pictures. Extend the table of values to show the next three terms in the sequence.
 - **a**) 6, 12, 18, 24, ...
 - **b**) 5, 9, 13, 17, ...
- **8. a)** Use a table of values to predict how many cubes you would need to build the 6th figure in this sequence.



b) Explain how the pattern rule works.

9. a) Use linking cubes to build these figures. Then build the next two figures in the sequence.



- **b)** Make a table of values to record the number of cubes you used to build each figure in part (a).
- c) Write a rule that describes the pattern.
- **10.** Asha has only 61 toothpicks. If she uses as many of these toothpicks as she can, what is the largest figure that she can build in the pattern?



11. Matthew and Suki are trying "The Cubic Challenge." They must build the 6th figure in the pattern shown below. Matthew says they will need a total of 216 cubes. Suki says they will need 108 cubes. Who is right? Explain.



Extending

12. This "staircase" sequence shows a growing pattern. Predict the total number of cubes you will need to build the 50th staircase. Show your thinking.

